2014/2015, week 6
The Ramsey model

Romer, Chapter 2.1 to 2.6

Background Ramsey model

- One of the main workhorses of macroeconomics
- Integration of
  - Empirical realism of the Solow Growth model and
  - Theoretical elegance of the classical model
- Basis for much other work

Background Ramsey model

- The Ramsey model borrows from the Solow Growth model
  - Aggregate production function
  - Not a real explanation of economic growth

Background Ramsey model

- The Ramsey model borrows from the classical model
  - Explanation of saving behaviour
  - Utility maximization as a basis to derive economic behaviour
  - Hence, the Ramsey model can be used for normative analysis

Background Ramsey model

- Everything has its price
  - The Ramsey model is technically much more complicated than the Solow Growth model or the classical Fisher model
  - Discussion in this course focuses on economic intuition, not on the mathematics

Background Ramsey model

- The Ramsey Model involves rational expectations
  - recall the Fisher model or the LCH model
- Rational expectations and endogenous saving behaviour combined:
  - the model cannot be solved recursively
  - current and future time periods need to be solved simultaneously
  - this course

Features of the Ramsey model

- Basically, developed for a closed economy
  - Without other institutions like a government, trade unions, social security, pension funds, banks
  - The only agents included are households and firms
- The representative household is infinitely-lived

Features of the Ramsey model

- Two interpretations are possible:
  - The household is assumed to live forever in order to get rid of complications that arise from connecting finite lifes with an economy that goes on forever
  - The household is viewed as a dynasty, an unending chain of generations that are connected to each other in family relationships

Features of the Ramsey model

- Intertemporal utility
  - $U = \int_{t=0}^{\infty} e^{-\rho t} u(C(t)) \frac{L(t)}{H} dt$
- $u\big(C(t)\big) = \frac{C(t)^{1-\theta}}{1-\theta} \qquad \theta > 0, \;\; \rho - n - (1-\theta)g > 0$
- Constant relative risk aversion utility function
  - Coefficient of relative risk aversion $\mathrm{CRRA} = -\frac{Cu''(C)}{u'(C)} = \theta$
  - Elasticity of intertemporal substitution $1/\theta$

Features of the Ramsey model

- Aggregate production function
  - $Y(t) = F\big(K(t), A(t)L(t)\big)$
- Labour and labour-augmenting technology grow at constant rates:
  - $\dot{L}(t)/L(t) = n$
  - $\dot{A}(t)/A(t) = g$

Features of the Ramsey model

- The representative household supplies labour to the firm and is also shareholder of the firm
  - This household receives labour income from the firm and also capital income of the firm
  - That is, the representative household receives all value added from the firm

Features of the Ramsey model

- Profit maximization by firms implies two first-order conditions:
  - $F_K(K, AL) = r + \delta$
  - $\delta = 0 \quad \rightarrow \; F_K(K, AL) = r$
  - $F_L(K, AL) = Aw$
- In intensive form:
  - $f'(k) = r$
  - $f(k) - f'(k)k = w$

Features of the Ramsey model

- Capital accumulation equation:
  - $\dot{k}(t) = f\big(k(t)\big) - c(t) - k(t)(n+g)$
- Compare this with the capital accumulation equation in the Solow growth model:
  - $\dot{k}(t) = sf(k(t)) - k(t)(\delta + n + g)$
- Only difference is nature of the saving rate (except for depreciation of capital)

Features of the Ramsey model

- Euler equation:
  - $\dot{C}(t) = C(t)\left(\frac{r(t)-\rho}{\theta}\right)$

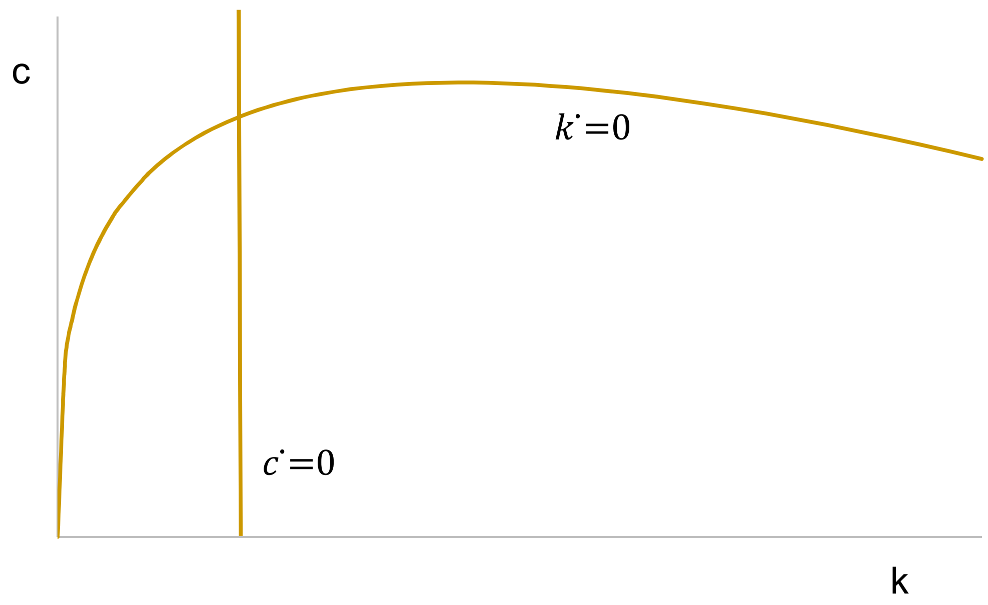
- As with the capital stock, we express consumption in intensive form:
  - $c(t) \equiv \frac{C(t)}{A(t)} \quad \rightarrow \quad \frac{\dot{c}(t)}{c(t)} = \frac{\dot{C}(t)}{C(t)} - \frac{\dot{A}(t)}{A(t)}$
  - $\frac{\dot{c}(t)}{c(t)} = \left(\frac{r(t)-\rho}{\theta}\right) - g \rightarrow$
  - $\dot{c}(t) = c(t)\left(\frac{r(t)-\rho-\theta g}{\theta}\right)$

Euler equation

- The differential equation for consumption is an old friend. Indeed, we can rewrite the optimality condition from the Fisher model into this differential equation
  - $\frac{C_2}{C_1} = \left(\frac{1+r}{1+\rho}\right)^{1/\theta}$
  - $ln\left(\frac{C_2}{C_1}\right) = 1/\theta(ln(1+r) - ln(1+\rho))$
  - $ln(1+x) \sim x$
  - $ln\left(1 + \frac{C_2 - C_1}{C_1}\right) \sim \frac{C_2 - C_1}{C_1} = 1/\theta(r - \rho)$

The dynamics of the model

The phase diagram I



The $\dot{k} = 0$ curve

- Compare this with the Solow Growth model
- There is a value of k for which c is at its maximum
- This is the Golden Rule level of k, which we also encountered in the Solow Growth model
- Smaller or larger values of k on the $\dot{k} = 0$ curve
  - imply less than maximal consumption per effective worker

The $\dot{c} = 0$ curve

- The $\dot{c} = 0$ curve is a vertical line:
  - There is only value for k that, through the interest rate r, implies stable consumption per effective worker

The steady state

- The steady state of the model is the intersection of the two curves
- As in the Solow model, the model exhibits stability:
  - Off steady state, the economy develops automatically to the steady state
  - Saddlepoint stability (see below)

The steady state

- The value for k corresponding to the steady state is called the Modified Golden Rule of capital accumulation
- The Modified Golden Rule level for k is always smaller than the Golden Rule level of k
- Recall Golden Rule condition:
  - $f'(k_{GR}) = n + g$

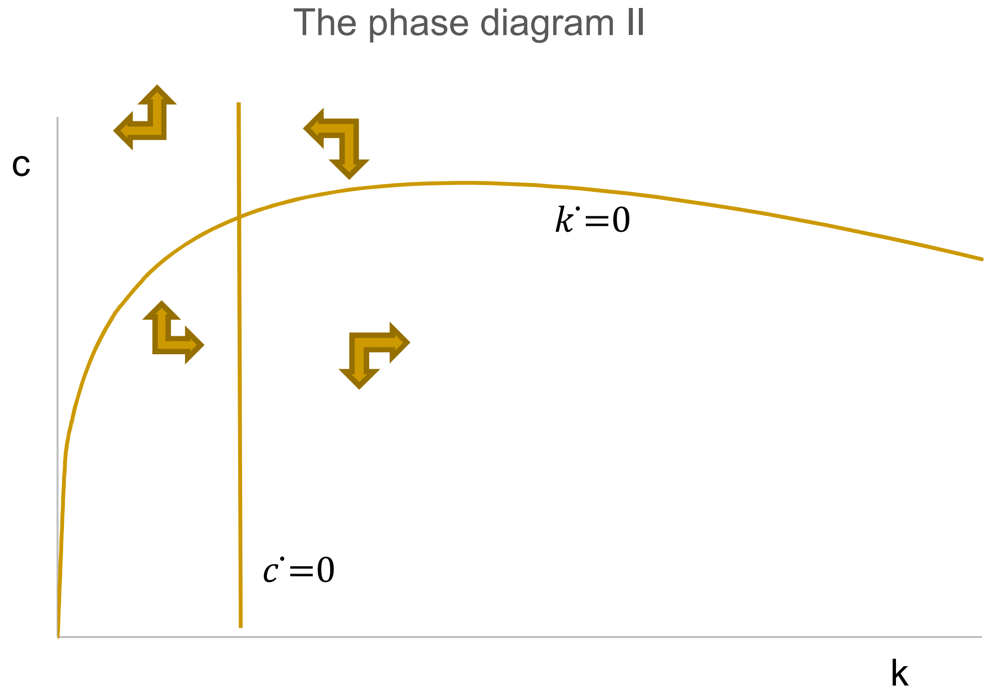
The steady state

- Steady-state condition:
  - $\dot{c}(t) = c(t)\left(\frac{r(t)-\rho-\theta g}{\theta}\right)$
  - $f'(k) = r$
  - $\frac{\dot{c}(t)}{c(t)} = 0 \rightarrow f'(k_{MGR}) = \rho + \theta g$
- Now, $k_{MGR} < k_{GR}$ if $f'(k_{MGR}) > f'(k_{GR})$, which implies
  - $\rho + \theta g > n + g$ or
  - $\rho - n - (1-\theta)g > 0$

Intuition for MGR result

- Households do not choose the level of k that corresponds to the Golden Rule
- From the perspective of the Modified Golden Rule, they would achieve higher steady-state consumption
- They would also have to first reduce consumption in order to accumulate the capital needed to reach the Golden Rule
- As the transition costs dominate the steady-state gains, the Golden Rule is suboptimal

The dynamics of the model



Saddle path

- The equations for $\dot{c} = 0$ and for $\dot{k} = 0$ describe the evolution of the economy for given starting values $c = c_0$ and $k = k_0$
- k_0 is a given (the capital stock is a predetermined variable)
- c_0 is free (consumption can be adjusted immediately)
- c_0 is calculated such that the model ends up in the steady state
- The value for c_0 that achieves this is unique:
  - Saddle point stability

Saddle path

- Romer, Figure 2.4, p. 61

- Too high value for c_0:
  - Ultimately, the capital stock will be zero, forcing consumption to zero, which conflicts with intertemporal utility maximization

- Too low value for c_0:
  - Ultimately, the capital stock will be higher than its Golden Rule level
  - This corresponds to dynamic inefficiency
  - Hence, consumption is less than what it could be, which conflicts with intertemporal utility maximization

Optimality

- First Welfare Theorem:
  - If markets are competitive
  - If markets are complete
  - If there are no externalities
  - The decentralized equilibrium is Pareto–efficient

- Pareto-efficiency:
  - It is impossible to make anyone better off without making someone else worse off

Optimality

- Popularity of the Ramsey model
- But, assume dictatorial regime
- Note the if conditions
  - If markets are competitive
  - If markets are complete
  - If there are no externalities

Balanced growth

- Output, the capital stock, consumption, saving (=investment) grow at rate $n + g$ on the balanced growth path
- Output and consumption per worker grow at rate g on the balanced growth path
- Output and consumption per effective worker are constant along the balanced growth path

A fall in the discount rate ρ

- Closest to the increase in the saving rate, which is exogenous in the Solow growth model
- The change is assumed unexpected
- Romer, Figure 2.6, p. 67
- Recall the capital stock is predetermined and consumption can be adjusted immediately
- New steady state implies higher capital stock and higher consumption
- Consumption will go down immediately and will increase only after some time