2015/2016, week 6 **The Ramsey model**

Romer, Chapter 2.1 to 2.6

Background for the Ramsey model

One of the main workhorses of macroeconomics

- Integration of
 - Empirical realism of the Solow Growth model and
 - Theoretical elegance of the classical model
- Basis for much other work

- The Ramsey model borrows from the Solow Growth model
 - Aggregate production function
 - Not a real explanation of economic growth

- The Ramsey model borrows from the classical model
 - Explanation of saving behaviour
 - Utility maximization as a basis to derive economic behaviour
 - Hence, the Ramsey model can be used for normative analysis

Everything has its price

- The Ramsey model is technically much more complicated than the Solow Growth model or the classical Fisher model
- Discussion in this course focuses on economic intuition, not on the mathematics

- The Ramsey Model involves rational expectations
 - recall the Fisher model or the LCH model
- Rational expectations and endogenous saving behaviour combined:
 - the model cannot be solved recursively
 - current and future time periods need to be solved simultaneously
 - this course

Basically, developed for a closed economy

- Without other institutions like a government, trade unions, social security, pension funds, banks
- The only agents included are households and firms
- The representative household is infinitelylived

Two interpretations are possible:

- The household is assumed to live forever in order to get rid of complications that arise from connecting finite lifes with an economy that goes on forever
- The household is viewed as a dynasty, an unending chain of generations that are connected to each other in family relationships

Intertemporal utility

$$\square U = \int_{t=0}^{\infty} e^{-\rho t} u(C(t)) \frac{L(t)}{H} dt$$

$$u(C(t)) = \frac{C(t)^{1-\theta}}{1-\theta} \quad \theta > 0, \ \rho - n - (1-\theta)g > 0$$

- Constant relative risk aversion utility function
 - Coefficient of relative risk aversion $CRRA = -\frac{Cu''(C)}{u'(C)} = \theta$
 - Elasticity of intertemporal substitution $1/\theta$

- Aggregate production function
 Y(t) = F(K(t), A(t)L(t))
- Labour and labour-augmenting technology grow at constant rates:

$$\Box \quad \dot{L}(t)/L(t) = n$$

 $\Box \dot{A}(t)/A(t) = g$

- The representative household supplies labour to the firm and is also shareholder of the firm
 - This household receives labour income from the firm and also capital income of the firm
 - That is, the representative household receives all value added from the firm

- Profit maximization by firms implies two first-order conditions:
 - $\square F_K(K, AL) = r + \delta$

$$\delta = 0 \quad \rightarrow F_K(K, AL) = r$$

 $\Box \quad F_L(K,AL) = Aw$

□ In intensive form:

$$f'(k) = r$$

$$f(k) - f'(k)k = w$$

Capital accumulation equation:

$$\dot{k}(t) = f(k(t)) - c(t) - k(t)(n+g)$$

 Compare this with the capital accumulation equation in the Solow growth model:

$$\hat{k}(t) = sf(k(t)) - k(t)(\delta + n + g)$$

Only difference is nature of the saving rate (except for depreciation of capital)

• Euler equation:

$$\Box \quad \dot{C}(t) = C(t) \left(\frac{r(t) - \rho}{\theta}\right)$$

As with the capital stock, we express consumption in intensive form:

$$c(t) \equiv \frac{C(t)}{A(t)} \rightarrow \frac{\dot{c}(t)}{c(t)} = \frac{\dot{C}(t)}{C(t)} - \frac{\dot{A}(t)}{A(t)}$$
$$\frac{\dot{c}(t)}{c(t)} = \left(\frac{r(t) - \rho}{\theta}\right) - g \rightarrow$$
$$\dot{c}(t) = c(t) \left(\frac{r(t) - \rho - \theta g}{\theta}\right)$$

Euler equation

 The differential equation for consumption is an old friend. Indeed, we can rewrite the optimality condition from the Fisher model into this differential equation

$$\Box \quad \frac{C_2}{C_1} = \left(\frac{1+r}{1+\rho}\right)^{1/\theta}$$

$$\square ln\left(\frac{C_2}{C_1}\right) = 1/\theta(ln(1+r) - ln(1+\rho))$$

$$\Box ln(1+x) \sim x$$

$$\Box \ ln\left(1 + \frac{C_2 - C_1}{C_1}\right) \sim \frac{C_2 - C_1}{C_1} = 1/\theta(r - \rho)$$

The dynamics of the model



The $\dot{k} = 0$ curve

- Compare this with the Solow Growth model
- There is a value of k for which c is at its maximum
- This is the Golden Rule level of k, which we also encountered in the Solow Growth model
- Smaller or larger values of k on the $\dot{k} = 0$ curve
 - imply less than maximal consumption per effective worker

The $\dot{c} = 0$ curve

- The $\dot{c} = 0$ curve is a vertical line:
 - There is only value for k that, through the interest rate r, implies stable consumption per effective worker

The steady state

- The steady state of the model is the intersection of the two curves
- Output and consumption per effective worker are constant along the balanced growth path
- Output, the capital stock, consumption, saving (=investment) grow at rate n + g on the balanced growth path
- Output and consumption per worker grow at rate g on the balanced growth path

The steady state

- As in the Solow model, the Ramsey model exhibits stability:
 - Off steady state, the economy develops automatically to the steady state
 - This occurs along the saddle path

The dynamics of the model



A fall in the discount rate ρ

- Closest to the increase in the saving rate, which is exogenous in the Solow growth model
- The change is assumed unexpected
- Romer, Figure 2.6, p. 67
- Recall the capital stock is predetermined and consumption can be adjusted immediately
- New steady state implies higher capital stock and higher consumption
- Consumption will go down immediately and will increase only after some time

A fall in the discount rate ρ

- The capital stock per effective worker expands along the transitional path
- What will happen to output per effective worker in the transition to the new steady state?
- What will happen to the interest rate in the transition to the new steady state?
- What will happen to the effective wage rate (price of effective labour) in the transition to the new steady state?

The steady state

- The value for k corresponding to the steady state is called the Modified Golden Rule of capital accumulation
- The Modified Golden Rule level for k is always smaller than the Golden Rule level of k
- Recall Golden Rule condition:

 $\Box f'(k_{GR}) = n + g$

The steady state

• Steady-state condition:

$$\dot{c}(t) = c(t) \left(\frac{r(t) - \rho - \theta g}{\theta}\right)$$

$$f'(k) = r$$

$$\frac{\dot{c}(t)}{c(t)} = 0 \rightarrow f'(k_{MGR}) = \rho + \theta g$$

• Now, $k_{MGR} < k_{GR}$ if $f'(k_{MGR}) > f'(k_{GR})$, which implies

•
$$\rho + \theta g > n + g$$
 or

 $\bullet \rho - n - (1 - \theta)g > 0$

Intuition for MGR result

- Households do not choose the level of k that corresponds to the Golden Rule
- From the perspective of the Modified Golden Rule, they would achieve higher steady-state consumption
- They would also have to first reduce consumption in order to accumulate the capital needed to reach the Golden Rule
- As the transition costs dominate the steady-state gains, the Golden Rule is suboptimal

Optimality

First Welfare Theorem:

- If markets are competitive
- If markets are complete
- □ If there are no externalities
- The decentralized equilibrium is Pareto-efficient

Pareto-efficiency:

 It is impossible to make anyone better off without making someone else worse off

Optimality

- Popularity of the Ramsey model
- But, assume dictatorial regime
- Note the if conditions
 - □ If markets are competitive
 - □ If markets are complete
 - □ If there are no externalities

- From a closed economy model to a small open economy model
 - or a multi-country model
- Inclusion of a government that provides public consumption or investment goods
- Inclusion of a labour-leisure decision
- Inclusion of taxes and subsidies
 - On labour income, capital income and consumption
 - On saving, investment, education, research and development regime

- Focus on international trade and international investment
- Focus on changing demographics
- Inclusion of the environment and taxes on pollution by firms and households
 - or a multi-country model

- Extension to a world in which one or more exogenous variables are stochastic
- Stochastic equity returns:
 - focus on the tradeoff between investment in equity and in bonds and the equity premium
- Stochastic inflation:
 - focus on the tradeoff between nominal bonds and inflation-linked bonds and the inflation risk premium
- Stochastic job markets:
 - □ focus on the effects of unemployment insurance

- Recall the assumptions of the Ramsey model
 - Fully rational households (and firms)
 - The rate of labour-saving technological progress is not explained within the model
 - Households form dynasties and are intergenerationally connected (Ricardian equivalence)