Paying for the Ageing Crisis: Who, How and When?

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Abstract

The prospect of population ageing renders fiscal policies in many countries unsustainable. Policy reforms therefore seem to be inevitable. The literature is almost silent however about *which policy instrument* should be used for this purpose, *when* reforms should be made and *which generations* would be mostly affected by these policy reforms. This paper explores these questions. It adopts a model distinguishing between different generations (an old, a young and future generations), between four types of government policy instruments (labor income taxes, transfers to the elderly, consumption of rival public goods and consumption of pure public goods), and between two types of demographic shocks (a longevity boost and a fertility bust). It concludes, first, that in general the government should implement reforms in all four instruments. The stronger is population ageing, the more the government should rely on cutting transfers to the elderly and the less it should rely on raising labor income taxes. Second, the reforms should be made immediately when information about future population ageing becomes available. Thirdly, although these policy reforms would affect all generations, young and future generations would in general be burdened most. In addition, the paper derives that in general the public debt should be decumulated in anticipation of ageing. Only if ageing stems from a fertility bust and the consumption of pure public goods is sufficiently high, will optimal policy reforms imply the opposite.

Keywords: Population ageing, Tax smoothing, Public debt

JEL Codes: H21, H40, H60

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1 Introduction

During the nineties, it became increasingly known that population ageing will have far-reaching implications. In the majority of industrialized countries, populations will become dramatically older in the decades to come: the number of elderly people will more or less double relative to the number of people of working age. Since a large part of the institutions for social security and health care is PAYG-financed, this means a huge amount of implicit debt that in most cases is not compensated by a sufficiently large negative statutory debt. What is more, the statutory public debt is positive in many countries, partly as a result of the current economic crisis.

Since then a complete new branch of industry has emerged that aims to quantify the budgetary consequences of ageing. In the early nineties, Blanchard *et al.* (1990) were the first to do so for a large number of industrialized countries. Auerbach *et al.* (1999) did a similar exercise, but more explicitly from the angle of generational accounting. Since then, a number of updates and extensions have been made, both for individual countries (*e.g.* Cardarelli *et al.* (2000), Faruqee and Mühleisen (2001), Congressional Budget Office (2003), Auerbach *et al.* (2004), Van Ewijk *et al.* (2006), Smid *et al.* (2014)) and for groups of countries (Gokhale and Raffelhüschen (2000), Dang *et al.* (2001), Balassone *et al.* (2009), European Commission (2012)). Börsch-Supan *et al.* (2006) and Fehr *et al.* (2008) consider ageing in different regions in the world and their interactions. It is fair to say that these studies often focus more on fiscal sustainability than on ageing per se. Indeed, other trends like globalization, climate change or the depletion of natural resources also affect fiscal sustainability (Heller 2003). Even economic growth may worsen fiscal sustainability (Andersen and Pedersen 2006). Generally, ageing is considered the most pervasive trend, however. Most studies abstract from any feedback from public debt upon the primary surplus, but there are exemptions (Bohn 1998).

Strangely enough, only a few studies are complemented by an analysis of the most appropriate way to handle the ageing problem. Many studies calculate the effects of typical policy reforms, such as reforming social security (*e.g.* Kotlikoff *et al.* (1999), De Nardi *et al.* (1999), Beetsma *et al.* (2003)) or reforming the health care system (Roig 2006). Studies that compare different reform strategies on their budgetary, economic and welfare effects are few, however. Balassone *et al.* (2009) explores the question whether a pre-funding strategy is more equitable than a gradual fiscal adjustment and its conclusions are similar to the conclusions in this paper. Balassone *et al.* (2009) leave aside the question which policy mix would be the optimal response to ageing, however.

This paper aims to fill this gap. In particular, it analyzes three questions: *when* should the ageing bill be paid, now or in the future? *who* should pay, current working generations, current retired generations or future generations? and *how* should the bill be paid, by tax increases or expenditure reductions? Insight into these questions is of fundamental importance. Policy reforms that try to tackle the ageing problem usually raise serious political opposition (Boeri *et al.* (2002)). In order to survive the political debate, policy reforms should probably be very balanced on all three aspects.

The paper adopts a model that captures not more than the elements that are essential for our analysis. The model is deterministic, contains two overlapping generations of households and describes a small open economy (factor prices are given). Households consume what they earn, but do make a labour supply decision. The government in the model produces rival and pure public consumption goods and spends on transfers to the elderly. It levies taxes on the labour income of young generations. Population ageing is modelled in two scenarios. In the longevity boost scenario the old generation grows in size, whereas in the fertility bust scenario the young cohort shrinks in size. The two scenarios have in common a permanent increase in the old-age dependency ratio (the size of the old cohort relative to that of the young cohort), but differ with respect to population size.

The paper draws five sets of conclusions. The first is quite obvious: it tells us that the government should act immediately when information about future population ageing arrives in the economy. In general, the prospect of population ageing changes optimal policies in all dimensions. The second set of conclusions is more novel. In particular, under restrictive conditions on the social discount rate and the rate of labor productivity growth, Barro's (1979) result that optimal tax policies exhibit the smoothing property carries over to transfer policies and rival public goods policies. Imposing more general conditions, smoothing will no longer be optimal however for the cases of optimal tax policies, optimal rival public consumption policies and optimal transfer policies. Moreover, the time profiles of optimal policies will then no longer converge; the differences relate to the intensity and to the form of population ageing. Thirdly, the case of pure public goods policies is entirely different. The optimal time profile of pure public goods consumption depends on the evolution of the size of the population. Hence, the scenarios of a longevity boost and of a fertility bust are each other's opposites with respect to optimal pure public goods policies. Fourthly, both the currently old and young generation and the future generations are hurt by the reforms, but the effect on the currently old will be in general somewhat more modest. The reason is that the currently old escape completely the effect of a higher tax burden and partly the effect of a cut in rival public consumption. The size of this imbalance depends on the intensity of population ageing: the higher the increase in the old-age dependency ratio, the lower will be the contribution of a tax increase to the government's implicit debt and the smaller will be the imbalance between the currently old generation and the other generations. However, in case ageing takes the form of a longevity boost, the imbalance will be smaller and may even change sign. The reason is that the longevity boost results in an increase in future public consumption from which all generations will benefit except the currently old. Fifthly, policy reforms imply in general a reduction of the public debt in anticipation of future population ageing, irrespective its form, *i.e.* a longevity boost or fertility bust. The size of the debt reduction is not a given, but is different for reforms in different policy instruments. However, in case ageing takes the form of a fertility bust and the consumption of pure public goods is relatively large, the opposite effect may occur: public debt is then increased in anticipation of higher primary balances which are due to cuts in future pure public consumption. Although this case may be less relevant empirically, we cannot exclude it a priori. This underscores that the evolution of public debt should be treated not as a target, but as the outcome of a process of optimal policy reform.[1]

Several papers have questioned the optimality of tax smoothing in economies that differ from the one assumed in Barro (1979). Tax smoothing is shown not to apply if the elasticity of labour supply varies over time (Aschauer (1988)), if the tax applies to capital as well (Chari *et al.* (1994)), if the tax base varies over time (Andersen and Dogonowski (2004)) or if the distribution of relative skills in the economy changes over time (Werning (2007)). As far as we know, the present paper is the first however that extends the concept of tax smoothing to other types of government policies (*i.e.*, transfer policies, rival public goods policies and pure public goods policies), that characterizes the optimal combination of these four types of government policies and that elaborates the implications for the development over time of the public debt and for the optimal balance between generations, both for the four types of government policies distinguished and for their optimal combination.

Caveats are also in order. Our analysis assumes a small open economy. It is therefore more applicable to one particular country that is hit by an idiosyncratic demographic shock than to a large group of industrialized countries that are hit by a common demographic shock and that respond (or anticipate) in a coordinated way. What pleads for the assumption of a small open economy is, first, that demographic projections for different countries differ in the timing and the intensity of population ageing and, second, that the policies investigated here belong to the domain of national authorities.

In addition, our paper assumes that demographic processes are exogenous to economic policies. This assumption is questionable, in particular in the long run. Empirical evidence indicates that the provision of public pensions may reduce fertility (*e.g.*, Cigno *et al.* (2003a) and

[1] A popular device involves that the public debt to GDP ratio should achieve a certain level at some point in time and derives from that the adjustment in some policy instrument (Dang *et al.* (2001), European Commission (2006)). The results in this paper demonstrate that this approach is not very useful. Even in the highly impropable case that the *ad hoc* assumption that is made on the level of the public debt ratio happens to be optimal in case a typical policy instrument is adjusted, it will definitively be wrong in case changes in other instruments are analysed.

Zhang and Zhang (2004)) and that the government can change fertility by appropriate transfer policies (Cigno *et al.* (2003b)). Similarly, there is empirical evidence that medical-technological progress is partly responsible for the increase in life expectancy during the last few decades (Cutler *et al.* (2006)). Although the role of these mechanisms is heavily debated, the evidence seems sufficiently strong to suggest that our analysis is incomplete. Given this, we regard our analysis only as a first step, but one that does yield some interesting new insights, however.

Furthermore, our approach of a stylized model is not the best in all respects. To be sure, it allows us to study economic policies on a more fundamental, qualitative level without getting lost in institutional details. For the purpose of assessing the effects of typical policy reforms, however, this approach seems inadequate. Indeed, for such kind of policy analysis one would benefit more from using models with many generations and a more detailed modelling of policy institutions, like in Kotlikoff *et al.* (1999), De Nardi *et al.* (1999), Beetsma *et al.* (2003) or Fehr *et al.* (2013).

The structure of the rest of the paper is as follows. Section 2 sets up our model of the ageing problem. Section 3 disucsses our two demographic shocks and the concept of (un-)sustainable debt. Section 4 explores the smoothing properties of the four types of government policies distinguished in this paper. Sections 5 and 6 then explore optimal policies, section 5 for the more general case and section 6 for a more specific case that allows us to solve the model analytically. Section 7 derives the implications of optimal policies for the balance between generations, whereas section 8 does the same for the evolution of the public debt in anticipation of future population ageing. Section 9 ends with some concluding remarks.

2 A model of the ageing problem

We use two periods to describe the life cycle of households. In the first period of their lives, households are young and supply labour. In the second and last period, they are old and retired. Households attach utility to consumption in both periods of their life cycle and to labour in the first period (negatively).

Although extremely stylized, a two-period setup is sufficient to model the key features of an ageing society. We consider the first period of the life cycle as referring to the period in between the age of 20 and the age of 50; the second period of the life cyle refers to the period in between the ages of 50 and 80. This corresponds only roughly to economic reality: although the age of 20 is not that different from the age at which people enter the labour market and the age of 80 is not that different from the average length of life, the age of 50, at which people retire in our model, deviates strongly from the average retirement age, which recently has been increasing in some countries. The implication of this is that our model is not very well equipped to make quantitative predictions.

Taxes, transfers and public consumption services can easily be fit into this model. From the generational accounting literature we know that net transfers from the public sector to households are generally positive in the second stage of their lives; in the first stage, these net transfers are generally negative. Our model mimics this pattern of net transfers by assuming that public consumption services benefit all generations equally, that transfers are made only to old generations and that taxes are due only by working generations. We deliberately talk about transfers rather than income transfers, as our definition of transfers includes also transfers in kind. Here, we think of health care services and long-term care services, the bulk of which is consumed by the elderly. Importantly, we will distinguish two types of public goods. Pure public goods conform to the standard Samuelson (1954) definition: they are non-rival and non-excludable. Rival public goods comprise private goods that are publicly provided and public goods that are subject to congestion. As argued in Barro (1990), many public goods such as highways, courts, water and sewer systems, national defense and police, may be subject to congestion: for these public goods, population growth diminishes the quality of a given amount of goods.

The government levies taxes on labour income only. Lumpsum taxes are absent from the model, an assumption that is easy to motivate. First, it is unclear whether taxes can really be distortionary. Indeed, even a tax unrelated to any economic activity can be avoided by emigrating from the country. Second, we want to focus on the main items on the government balance. Lumpsum taxes are, in terms of revenues raised, relatively unimportant everywhere in the industrialized world.

The same does obviously not hold true for consumption taxes. Yet, consumption taxes are absent as well from the model. In our model, a combination of the labour income tax (imposed on the working generation) and transfers (to the elderly) would be superior to a consumption tax, which does not differentiate between generations. Hence, if we would extend the set of government policies with a consumption tax, this tax would never be used. We think that extending our model with heterogeneity within cohorts would allow a role for the consumption tax, but consider such an extension well beyond the scope of this paper.

We model population ageing as a permanent increase of the old-age dependency ratio, defined as the size of the old cohort relative to the size of the young cohort. This can be due to a permanent increase in the survival rate, which increases the size of the old cohort, but leaves unchanged the size of the young cohort. In this case, the increase in the old-age dependency ratio corresponds with a permanent expansion of the population. The increase of the old-age dependency ratio can also stem from a temporary bust of the fertility rate, combined with a permanent increase of the survival rate in the period thereafter. In this second scenario, the increase in the old-age dependency ratio is accompanied with a permanent decline of the population. The two scenarios are counterparts. By calibration, we can get exactly the same increase of the old-age dependency ratio under the two scenarios. The scenarios have opposite implications for population size, however. Hence, our model can mimic the demographics of the 22 OECD countries that are covered in the OECD report on ageing (Dang *et al.* 2001)). In particular, it can match those cases in which ageing is expected to yield an expansion of the population in the period up to 2050, like Australia, Norway and the US. It can also mimic the cases in which ageing combines with a shrinking population in the same period, like Italy, Japan and the Czech Republic. In addition, a specific combination of the two scenarios would yield population ageing without any change in the size of the population, which would fit cases like Korea, Belgium and the UK.

2.1 The demographic structure

The demographic structure in this two-period model is fully determined by the structure in the previous period and the fertility and survival rate. More precisely, the size of the young cohort in period t, n_t^y, is determined by the size of the predecessing cohort and the fertility rate f_t: $n_t^y = f_t n_{t-1}^y$. The size of the old cohort, n_t^o, follows from multiplying the size of the same cohort one period earlier with the survival rate ζ_t: $n_t^o = \zeta_t n_{t-1}^y$. The old-age dependency ratio, defined as n_t^o/n_t^y, therefore equals the ratio between the survival rate and the fertility rate: ζ_t/f_t.

2.2 Households

The intertemporal utility function of households consists of three sub-utility functions: one for a composite of private consumption goods and leisure, one for rival public goods and one for pure public goods. Intertemporal utility for the household who is born in period t thus reads as follows:

$$
\begin{aligned}
u_t = {} & uc\left(c_t^y - 1/2\beta l_t^2\right) + \frac{\zeta_{t+1}}{1+r} uc\left(c_{t+1}^o\right) \\
& + ub^r\left(b_t^r\right) + \frac{\zeta_{t+1}}{1+r} ub^r\left(b_{t+1}^r\right) \\
& + ub^u\left(b_t^u\right) + \frac{\zeta_{t+1}}{1+r} ub^u\left(b_{t+1}^u\right) \qquad (2.1)
\end{aligned}
$$

Here, u, c^y, c^o and l are used to denote intertemporal utility, private consumption during the first and second period of the life cycle and labour supply respectively. b^r denotes the volume of rival public goods, b^u the volume of pure public goods. Both types of goods benefit both young and old generations. uc, ub^r and ub^u are sub-utility functions, that have the standard properties:

$uc'(.) > 0$, $uc''(.) < 0$, $ub^{r'}(.) > 0$, $ub^{r''}(.) < 0$, $ub^{u'}(.) > 0$ and $ub^{u''}(.) < 0$. $r > 0$ denotes the interest rate which we assume constant. ζ_{t+1} denotes the survival rate of the cohort that is born in period t.

We assume that the capital market is closed for households. This is an extreme assumption, but no more extreme than the neoclassical assumption of perfect capital markets on which households can trade costlessly, not being hindered by liquidity, borrowing and short-selling constraints. Given the assumption that households cannot borrow nor lend, first-period consumption equals after-tax labour income and second-period consumption equals the (public) pension p,

$$c_t^y = w_t(1-\tau_t)l_t \tag{2.2}$$

$$c_t^o = p_t \tag{2.3}$$

where w denotes the wage rate and τ the rate of labour income taxation. The wage rate grows at constant rate g.

Maximization of equation (2.1) subject to the constraint (2.2) yields the following expression for labour supply:[2]

$$l_t = \frac{1}{\beta} w_t(1-\tau_t) \tag{2.4}$$

As equation (2.4) shows, the household's financial wealth does not enter the labour supply equation and labour supply is strictly increasing in the after-tax wage rate.

With the help of the labour supply equation (2.4), the following expression for household t's indirect utility can be derived:

$$\begin{aligned} v_t = {} & uc\left(\frac{w_t^2(1-\tau_t)^2}{2\beta}\right) + \frac{\zeta_{t+1}}{1+r} uc\left(p_{t+1}\right) \\ & + ub^r\left(b_t^r\right) + \frac{\zeta_{t+1}}{1+r} ub^r\left(b_{t+1}^r\right) \\ & + ub^u\left(b_t^u\right) + \frac{\zeta_{t+1}}{1+r} ub^u\left(b_{t+1}^u\right) \end{aligned} \tag{2.5}$$

2.3 The government

The government is assumed to maximize a social welfare function. Social welfare, denoted as W, adds up the indirect utility indices of all generations involved. It sufices to define the social welfare function for t=1, the period in which the government receives the information that there

[2] We do not consider explicitly the time constraint that labour supply should not exceed time available for labour and leisure. We assume that this constraint is never binding by appropriate calibration.

will be an ageing shock in the next period (longevity boost or fertility bust):

$$W_1 = \sum_{i=0}^{\infty} \frac{n_i^y v_i}{(1+\delta)^{i-1}} \tag{2.6}$$

Equation (2.6) expresses that in determining optimal government policies, the government takes into account the interests of all current and future generations, including those of the currently old. $\delta \geq 0$ is used to denote the social discount rate, which may be equal or unequal to the interest rate r.

The maximization problem is subject to the intertemporal government budget constraint. To derive this constraint, we write down the debt accumulation equation. This equation describes how the public debt develops over time given the time pattern of taxes and (primary) expenditures. Let us denote the public debt at the end of period t-1 as D_{t-1}. Further, let us denote expenditure on rival public goods, expenditure on pure public goods, expenditure on transfers and tax revenues during period t as B_t^r, B_t^u, P_t and T_t respectively. Finally, let us assume that these income flows occur at the beginning of period t. The debt accumulation equation then reads as follows:

$$D_t = (D_{t-1} + B_t^r + B_t^u + P_t - T_t)(1+r) \tag{2.7}$$

We impose a solvency condition to the public sector. Technically speaking, the government is required to have eliminated the public debt in present-value terms at the end of her planning horizon:

$$lim_{N \to \infty} \frac{D_N}{(1+r)^N} = 0 \tag{2.8}$$

Combining the debt accumulation equation (2.7) and the solvency condition (2.8), we arrive at the intertemporal government budget constraint:

$$\sum_{t=1}^{\infty} \frac{T_t}{(1+r)^{t-1}} - \sum_{t=1}^{\infty} \frac{B_t^r}{(1+r)^{t-1}} - \sum_{t=1}^{\infty} \frac{B_t^u}{(1+r)^{t-1}} - \sum_{t=1}^{\infty} \frac{P_t}{(1+r)^{t-1}} - D_0 = 0 \tag{2.9}$$

According to this constraint, the public debt at the beginning of the planning horizon should be redeemed by primary budget surpluses, either in the near or in the distant future.

Tax revenues equal the tax rate times the tax base, which is labour income:

$$T_t = \tau_t w_t l_t n_t^y = \frac{w^2}{\beta} n_t^y \tau_t (1 - \tau_t) \tag{2.10}$$

The second part of this equation makes use of the labour supply equation (2.4).

For expenditure on transfers, on pure public goods, and on rival public goods, we have the following definitional equations:

$$P_t = p_t n_t^o \tag{2.11}$$

$$B_t^u = b_t^u \tag{2.12}$$

$$B_t^r = b_t^r n_t \tag{2.13}$$

where $n_t \equiv n_t^y + n_t^o$ denotes the size of the total population in period t.

Expenditure on transfers is proportional with the elderly population, as in our model only the elderly receive transfers (comprising public pensions and health care). Expenditure on pure public goods is not related to population size, reflecting the nonrivalrous nature of these goods. For the same reason, expenditure on rival public goods does relate to population size.

We adopt a normative approach to government policies and adopt the utilitarian criterion in formulating a social welfare function. There are alternative approaches like the median-voter approach to the determination of government policies as in Razin *et al.* (2002) or the Rawlsian social welfare function and we cannot claim that our approach is superior to such alternatives. However, we think the normative approach based on the utilitarian criterion is a useful starting point.

We can now proceed to define the problem of the government. This is to maximize social welfare, equation (2.6), subject to the intertemporal government budget constraint, equation (2.9). More concisely, the government maximizes the following Lagrangian with λ as Lagrange multiplier:

$$\mathscr{L}_1 = W_1 + \lambda \left[\sum_{i=1}^{\infty} \frac{T_i}{(1+r)^{i-1}} - \sum_{i=1}^{\infty} \frac{B_i^r}{(1+r)^{i-1}} - \sum_{i=1}^{\infty} \frac{B_i^u}{(1+r)^{i-1}} - \sum_{i=1}^{\infty} \frac{P_i}{(1+r)^{i-1}} - D_0 \right] \tag{2.14}$$

Below, we will elaborate optimal government policies. Before doing so, we discuss the two types of demographic shocks of which the effects will be studied: a longevity boost and a fertility bust.

3 Two scenarios: a longevity boost and a fertility bust

We model population ageing as a permanent increase in the old-age dependency ratio. This can be due to two types of shocks, both of which are relevant in the context of population ageing. The first is that of a longevity shock, a permanent increase of longevity. In this case, the period-2 survival rate ζ_2, which defines which part of the cohort born in period 1 survives into period 2, is higher than the previous cohort's survival rate; cohorts born after period 1 share the higher survival rate. The fertility rate remains unchanged at the value of unity. This longevity shock increases the old-age dependency ratio and expands the population in period 2. Thereafter, both the old-age dependency ratio and the size of the population remain constant. Figure 1 illustrates.

The second shock is a temporary bust in fertility. The fertility rate equals unity initially and falls to a lower level in period 2, f_2. Thereafter, the fertility rate is unity again. The survival rate increases to $1/f_2$ in period 3 and remains at that level thereafter. This fertility shock increases the old-age dependency ratio and reduces the population in period 2. After period 2, both the old-age dependency ratio and the size of the population remain constant. Figure 2 illustrates.

In figures 1 and 2, the blocks 'y' and 'o' correspond to the size of the young and old cohort respectively in different time periods. Displayed are time periods from t=-1 up to t=4; the periods t<-1 are equal to t=-1; the time periods t>4 equal t=4. The horizontal arrows refer to the fertility rate, the diagonal arrows to the survival rate. The figure clearly shows that before t=1, the economy is in steady state: the size and age structure of the population are unchanged over time. The long arrows indicate what changes relative to this steady state. In the case of a longevity boost, it is the survival rate that changes permanently in period t=2. In the case of a fertility bust, the fertility rate changes temporarily in period t=2 and the survival rate changes permanently in period t=3. The figure also shows that both cases feature a permanent increase in the size of the old cohort relative to the young cohort. In the case of a longevity boost, the population expands to a permanently higher level, whereas in the case of a fertility bust, the population shrinks to a permanently lower level.

[Figure 1 about here.]

[Figure 2 about here.]

Our modeling of population ageing is very stylized. It implies that age structure and size of the population change only from period t=1 to t=2. Taking a model period as a period of thirty years, this does very roughly correspond to the basic characteristic of population ageing in many countries, namely a permanent increase in the old-age dependency ratio, combined with an increasing, stable or falling population.

The following equation defines the implicit debt of the government, given that the population structure is stationary from period t=2 onwards:

$$\begin{aligned} G_0 = & \left(n_1 b_1^r + n_2 \sum_{t=2}^{\infty} \frac{b_t^r}{(1+r)^{t-1}} \right) + \left(b_1^u + \sum_{t=2}^{\infty} \frac{b_t^u}{(1+r)^{t-1}} \right) \\ & + \left(n_1^o p_1 + n_2^o \sum_{t=2}^{\infty} \frac{p_t}{(1+r)^{t-1}} \right) - \left(n_1^y \frac{w_t^2}{\beta} \tau_1 (1-\tau_1) + n_2^y \sum_{t=2}^{\infty} \frac{\frac{w_t^2}{\beta} \tau_t (1-\tau_t)}{(1+r)^{t-1}} \right) \end{aligned} \tag{3.1}$$

Initially, government policies are sustainable, *i.e.* $G_0 = -D_0$. Given the perspective of a stationary population, they maximize social welfare, subject to the intertemporal government budget constraint. Hence, b_1^r, b_1^u, p_1 and τ_1 in (3.1) reflect optimal policies that can be derived with the conditions that will be explored below. Then, in period 1 an ageing shock arrives (a longevity boost or a fertility bust). We assume that then public finances are no longer sustainable, but unsustainable, *i.e.* $G_0 > -D_0$. This accords with the situation in most European countries (see European Commission 2012).

Given the amount of statutory public debt which is inherited from the past, reducing the total public debt means reducing the implicit public debt. Stated alternatively, G_0 in equation (3.1) should be reduced to $-D_0$ in order to restore fiscal sustainability. The interesting question is how. Our analysis distinguishes four instruments of government policies that can be changed at two points in time. Optimal policies are defined as that combination of changes in the four policy instruments at these two points in time that ensures that social welfare is maximized. We will study the interactions between the four types of policies in sections 5 and 6. First, we will explore the optimal time profile of each of the four policy instruments in the next section, however.

4 Four flavours of smoothing policies

This section explores the properties of the four instruments of government policies. We start with rival public goods policies. Elaboration of the first-order condition for optimal rival public goods policies in period t yields the following equation:

$$\partial \mathscr{L}_1 / \partial b_t^r = 0 \qquad \longrightarrow \qquad \left(\frac{n_t^o \left(\frac{1+\delta}{1+r}\right) + n_t^y}{n_t} \right) \left(\frac{1+\delta}{1+r} \right)^{-(t-1)} u b^{r'} (b_t^r) = \lambda \tag{4.1}$$

Equation (4.1) shows two things. First, if there is no population ageing, the evolution of rival public consumption over time depends on the relationship between the social discount rate δ and the interest rate r. If the social discount rate is higher than the interest rate, it is optimal to decrease the consumption of rival public goods over time. If the social discount rate is lower, the opposite holds true. Only if the social discount rate and the interest rate coincide, will it be optimal to smooth the consumption of rival public goods over time. Second, ageing of the population has no effect upon the time profile of optimal rival goods consumption if the social discount rate and the interest rate are equal, but has an effect if the social discount rate and the interest rate deviate. In particular, if the social discount rate is higher than the interest rate, the jump in the old-age dependency ratio from period t=1 to t=2 increases the consumption of rival public goods, *i.e.* $b_2^r > b_1^r$. If the social discount rate is lower than the interest rate, it is optimal to let the consumption of rival goods decrease from period t=1 to t=2, *i.e.* $b_2^r < b_1^r$. The reason is that ageing affects the weights attached to periods t=1 and t=2 in the social welfare function. If $\delta > r$, an increase in the number of elderly in period t=2 increases the weight given to that period in the social welfare function, thereby making it optimal to have a large amount of rival public consumption in that period. If $\delta < r$, an increase in the number of elderly in period t=2 decreases the weight of this period in the social welfare function with the opposite effect upon rival public consumption. A decrease in the number of youngsters has the same effect as an increase in the number of elderly as it is the ratio of the two that determines the weight of the period in question in the social welfare function.

The equation for the optimal consumption of pure public goods is derived in a similar manner:

$$\partial \mathscr{L}_1/\partial b_t^u = 0 \qquad \longrightarrow \qquad \left(n_t^o(\frac{1+\delta}{1+r}) + n_t^y\right)\left(\frac{1+\delta}{1+r}\right)^{-(t-1)} ub^{u'}(b_t^u) = \lambda \tag{4.2}$$

Equation (4.2) is similar to equation (4.1) in one respect and very different in another one. The similarity concerns the relationship between optimal policies and the factor $(1+\delta)/(1+r)$ in the absence of population ageing. As in the case of rival public consumption policies, if there is no ageing of the population, the consumption of pure public goods should be decreased over time if the social discount rate is higher than the interest rate and vice versa; smoothing obtains only if the social discount rate and interest rate are equal. Very different is the effect of a change in the age structure of the population. In particular, if the population is expected to expand, as is the case in the longevity boost scenario, it is optimal to let the consumption of pure public goods increase from period t=1 to t=2, *i.e.* $b_2^u > b_1^u$. If the population is expected to shrink, as is the case in the fertility bust scenario, it is optimal to let the consumption of pure public goods decrease from period t=1 to t=2, *i.e.* $b_2^u < b_1^u$. Both effects occur irrespective the relation between δ and r. These results echo the peculiar feature of pure public goods: a change in population size changes their social benefits, but not their social costs.

The case of optimal transfer policies is derived similarly. The corresponding first-order condition reads as follows:

$$\partial \mathscr{L}_1/\partial p_t = 0 \qquad \longrightarrow \qquad \left(\frac{1+\delta}{1+r}\right)^{-(t-2)} uc^{'}(p_t) = \lambda \tag{4.3}$$

Like in the two previous cases, the higher is the social discount rate, the less is the weight given to future generations and the larger the rate of decline of optimal transfers over time. Again, constancy of transfers over time is only achieved if the social discount rate and interest rate are equal to each other. However, unlike the two previous cases, demographic factors play no role here. The reason is that, unlike the cases of rival and pure public consumption policies, in the case of transfer policies only one generation is involved.

The case of optimal tax policies yields a more complex first-order condition:

$$\partial \mathscr{L}_1/\partial \tau_t = 0 \qquad \longrightarrow \qquad \left(\frac{1+\delta}{1+r}\right)^{-(t-1)} uc^{'}\left(\frac{w_t^2(1-\tau_t)^2}{2\beta}\right)\left(\frac{1-\tau_t}{1-2\tau_t}\right) = \lambda \tag{4.4}$$

This equation tells us that optimal tax policies feature smoothing if the social discount rate equals the interest rate and the rate of labor productivity growth is zero. The more general case is different. However, due to the complexity of the first-order condition, we are unable to draw any conclusions regarding optimal tax policies. But we can make a step by arguing that for small tax rates, changes in τ_t have little effect on the factor $(1-\tau_t)/(1-2\tau_t)$, so that we can, as an approximation, neglect this factor. What can we then say? First, assuming zero labor productivity growth, if the social discount rate exceeds the interest rate, it would be optimal to let the tax rate increase over time. If the interest rate is higher than the social discount rate, the opposite holds true. Second, labor productivity growth in itself requires the tax rate to increase over time. As regards ageing of the population, we can say more. Even without making the approximation, we can conclude that ageing does not have any effect upon the optimal tax profile. The reason is the same as in the previous case. In the case of tax policies, only one generation is involved. This is what distinguishes tax and transfer policies from rival and pure public consumption policies, where ageing may have an effect upon optimal policies.

Next to the differences between the four first-order conditions, there is also an interesting common element. That is that when new information arrives about future changes in the population, the government should act immediately. Indeed, in general, both current and future policies depend on the future change in the population. Only by coincidence will optimal policies in period t=1 coincide with the policies that were considered optimal before the news about future demographic changes had arrived.

This section has made clear the role of the discount rate (relative to the interest rate) and the rate of labor productivity growth. As we have seen, the precise value of the discount rate is not always relevant. Indeed, the discount rate does not interfere with the increase in the old-age dependency ratio in two out of four cases. In addition, we have little to no information about what is an appropriate value for the social discount rate. In order to avoid drawing the wrong conclusions, we therefore leave this issue in the following and put δ equal to r. As to labor productivity growth, the precise value of g is not always relevant either. The rate of labor productivity growth does not interfere with ageing. It does affect the profile of tax rates over time, but this may be of little relevance, given that a number of economists expect future labor productivity growth to be fairly low. In the following, we will therefore put g equal to zero.

As can be seen by looking at the equations (4.1) to (4.4), these two assumptions simplify the first-order conditions. Apart from pure public goods policies, all types of government policies now exhibit smoothing. But there is more at stake. The assumptions δ=r and g=0, combined with the result established in section 3 that the population has constant size and age structure from period t=2 onwards, imply that the model now becomes completely stationary from period t=2 onwards. Hence, we can condense all future periods starting from period t=2 into one period. Let us denote this period henceforth as t=2+. Note that the t=2+ period is of infinite length, but has, due to discounting, finite weight.

Because the model is in steady state from period t=2 onwards, the intertemporal budget constraint of the government simplifies considerably:

$$
\begin{aligned}
-D_0 = & \left(n_1 b_1^r + n_{2+}\frac{b_{2+}^r}{r}\right) + \left(b_1^u + \frac{b_{2+}^u}{r}\right) \\
& + \left(n_1^o p_1 + n_{2+}^o \frac{p_{2+}}{r}\right) - \frac{w^2}{\beta}\left(n_1^y \tau_1(1-\tau_1) + n_{2+}^y \frac{\tau_{2+}(1-\tau_{2+})}{r}\right)
\end{aligned}
\tag{4.5}
$$

5 Optimal policy reform: the general case

In order to say more about optimal policies, we now combine the first-order conditions derived above for the four instruments of government policy.

Optimal policies in case four instruments are used simultaneously can be found by combining the first-order conditions presented in equations (4.4), (4.3), (4.2) and (4.1) under the assumptions δ=r and g=0. This gives rise to three optimality conditions. Each of the optimality conditions can be written as a linear relationship between the marginal changes in two instruments. Let us use the index CP to refer to optimally combined policies. The first condition and relationship apply to tax policies and rival public goods policies and are based on first-order conditions (4.4) and (4.1):

$$
\begin{aligned}
& ub^{r'}(b^r(\mathrm{CP})) = uc'\left(\frac{w^2(1-\tau(\mathrm{CP}))^2}{2\beta}\right)\left(\frac{1-\tau(\mathrm{CP})}{1-2\tau(\mathrm{CP})}\right) \quad \longrightarrow \\
& ub^{r''}(.)db^r(\mathrm{CP}) + \left(\frac{uc''(.)w^2(1-\tau(\mathrm{CP}))^2}{\beta(1-2\tau(\mathrm{CP}))} - \frac{uc'(.)}{(1-2\tau(\mathrm{CP}))^2}\right)d\tau(\mathrm{CP}) = 0
\end{aligned}
\tag{5.1}
$$

Note that we have made use of the smoothing property of both first-order conditions (4.4) and (4.1), so that time subscripts do not appear in condition (5.1).

The second optimality condition and relationship refer to transfer policies and rival public goods policies and combine (4.3) with (4.1):

$$
\begin{aligned}
& uc'(p(\mathrm{CP})) = ub^{r'}(b^r(\mathrm{CP})) \quad \longrightarrow \\
& dp(\mathrm{CP}) = \left(\frac{ub^{r''}(.)}{uc''(.)}\right)db^r(\mathrm{CP})
\end{aligned}
\tag{5.2}
$$

Finally, the third optimality condition follows from combining (4.2) and (4.1) and expresses the preference for pure public goods policies relative to rival public goods policies. As explained above, even when δ=r and g=0, population size enters condition (4.2). Hence, the third optimality condition distinguishes between periods t=1 and t=2+.

$$
\begin{aligned}
& n_t ub^{u'}(b_t^u(\mathrm{CP})) = ub^{r'}(b^r(\mathrm{CP})) \qquad t = 1, 2+ \quad \longrightarrow \\
& db_t^u(\mathrm{CP}) = \left(\frac{ub^{r''}(.)}{n_t ub^{u''}(.)}\right)db^r(\mathrm{CP}) - \left(\frac{ub^{u'}(.)}{n_t ub^{u''}(.)}\right)dn_t \qquad t = 1, 2+
\end{aligned}
\tag{5.3}
$$

Note that the first optimality condition, in equation (5.1), relates the variables $b^r(\mathrm{CP})$ and $\tau(\mathrm{CP})$ to one another. A second relationship between these two variables emerges when we elaborate the intertemporal government budget constraint. Recall equation (4.5) from section 2 and substitute optimal policies:

$$\begin{aligned} -D_0 = {} & Nb^r(\mathrm{CP}) + \left(b_1^u(\mathrm{CP}) + \frac{b_{2+}^u(\mathrm{CP})}{r}\right) + N^o p(\mathrm{CP}) \\ & - N^y \frac{w^2}{\beta}\tau(\mathrm{CP})(1-\tau(\mathrm{CP})) \end{aligned} \tag{5.4}$$

Here, we have used some shorthand notation. N stands for $n_1 + n_{2+}/r$, N^y for $n_1^y + n_{2+}^y/r$ and N^o for $n_1^o + n_{2+}^o/r$.

We totally differentiate this equation with respect to the policy instruments and the population variables. Note that $dn_1 = 0$ and that we have to distinguish between n_{2+}^y and n_{2+}^o in order to cover both the case of a longevity boost and that of a fertility bust:

$$\begin{aligned} & Ndb^r(\mathrm{CP}) + db_1^u(\mathrm{CP}) + \frac{1}{r}db_{2+}^u(\mathrm{CP}) + N^o dp(\mathrm{CP}) - N^y\frac{w^2}{\beta}(1-2\tau(\mathrm{CP}))d\tau(\mathrm{CP}) \\ & + \left(\frac{b^r(\mathrm{CP}) + p(\mathrm{CP})}{r}\right)dn_{2+}^o + \left(\frac{b^r(\mathrm{CP}) - \frac{w^2}{\beta}\tau(\mathrm{CP})(1-\tau(\mathrm{CP}))}{r}\right)dn_{2+}^y = 0 \end{aligned} \tag{5.5}$$

We elaborate this equation by substituting the expressions derived in equations (5.2) and (5.3), the latter for t=1 and t=2+. This gives rise to the following relationship between $b^r(\mathrm{CP})$ and $\tau(\mathrm{CP})$:

$$\begin{aligned} & \left(N + \frac{ub^{r\prime\prime}(.)}{n_1 ub^{u\prime\prime}(.)} + \frac{1}{r}\frac{ub^{r\prime\prime}(.)}{n_{2+}ub^{u\prime\prime}(.)} + N^o\frac{ub^{r\prime\prime}(.)}{uc^{\prime\prime}(.)}\right)db^r(\mathrm{CP}) \\ & -N^y\frac{w^2}{\beta}(1-2\tau(\mathrm{CP}))d\tau(\mathrm{CP}) \\ & + \left(\frac{b^r(\mathrm{CP}) + p(\mathrm{CP})}{r} - \frac{1}{r}\frac{ub^{u\prime}(.)}{n_{2+}ub^{u\prime\prime}(.)}\right)dn_{2+}^o \\ & + \left(\frac{b^r(\mathrm{CP}) - \frac{w^2}{\beta}\tau(\mathrm{CP})(1-\tau(\mathrm{CP}))}{r} - \frac{1}{r}\frac{ub^{u\prime}(.)}{n_{2+}ub^{u\prime\prime}(.)}\right)dn_{2+}^y = 0 \end{aligned} \tag{5.6}$$

Now, we have two equations, (5.1) and (5.6), in two unknowns, $db^r(\mathrm{CP})$ and $d\tau(\mathrm{CP})$. In general, there is a unique solution. Equation (5.1), which we will call the preference curve, is downward sloping, whereas equation (5.6), which we will call the budget constraint curve, is upward sloping. Note that the shape of the preference curve depends on the value of $1-2\tau(\mathrm{CP})$. Should $\tau(\mathrm{CP})$ be larger than 1/2, the downward sloping profile would not be guaranteed. However, optimality implies that the tax rate is strictly smaller than 1/2, *i.e.* we are on the left part of the Laffer curve (this can also be derived from equation (4.4)).

Figure 3 gives an illustration. Its first quadrant displays the preference curve and the budget constraint curve with $b^r(\mathrm{CP})$ on the y-axis and $\tau(\mathrm{CP})$ on the x-axis. It is drawn for the specific case that will be studied in the next section. More generally, the curves need not be quadratic, but their slopes will be as drawn: negative for the preference curve and positive for the budget constraint curve on the traject $\tau(\mathrm{CP}) \in [0, 1/2)$. The intersection of the two curves in the first quadrant of Figure 3 is denoted as A. This gives optimal policies ($b^r(\mathrm{CP})$, $\tau(\mathrm{CP})$). Equation (5.2), the relationship between $dp(\mathrm{CP})$ and $db^r(\mathrm{CP})$, is displayed in the second quadrant of Figure 3 . Next, equation (5.2) and equation (5.3) for t=1 can be combined to derive a relationship between $db_1^u(\mathrm{CP})$ and $dp(\mathrm{CP})$. This relationship is displayed in the third quadrant of Figure 3. Finally, we use equation (5.3) to derive a relationship between $db_{2+}^u(\mathrm{CP})$ and $db_1^u(\mathrm{CP})$. This relationship is displayed in the fourth quadrant of the figure.

The following results can now be obtained for the two ageing scenarios.

5.1 The longevity boost scenario

The longevity boost scenario features dn_{2+}^o>0 and dn_{2+}^y=0. It is easy to see that the demographic shift does not affect the preference curve (see equation (5.1)). It can be derived quite easily that the budget constraint curve shifts to the right (see equation (5.6)). Hence, the intersection of the two curves shifts to the right and downward, in Figure 3 from A to B. Hence, $\tau(\mathrm{CP})$ increases and $b^r(\mathrm{CP})$ decreases. Figure 3 also shows that optimal policy reform implies a decrease of $p(\mathrm{CP})$ and $b_1^u(\mathrm{CP})$. The effect upon $b_{2+}^u(\mathrm{CP})$ is ambiguous, however, due to the fact that the sustainability problem requires a decrease of $b_{2+}^u(\mathrm{CP})$, but the expansion of the population that features the longevity boost implies an increase. It then depends on the structure of preferences which of the two effects dominates.

5.2 The fertility bust scenario

The fertility bust scenario features dn_{2+}^y<0 and dn_{2+}^o=0. The discussion of the fertility bust scenario is similar to that of the longevity boost scenario, except that an additional assumption has to be made. That is that the coefficient of dn_{2+}^y in equation (5.6) is strictly negative. Loosely speaking, this means that tax revenues should be sufficiently larger than the spending on rival consumption goods. This seems a quite innocent assumption, given that tax revenues are usually used to finance not only spending on these goods, but on a number of other items as well.

Let us assume that this condition is met. The effects of a fertility bust are then the same as those of a longevity boost, except for the change in the future consumption of the pure public good. Now, the effect upon $b_{2+}^u(\mathrm{CP})$ is unambiguously negative, reflecting that both the sustainability problem and the decline of the population that results from the fertility bust call for a decrease of $b^u_{2+}(\mathrm{CP})$.

6 Optimal policy reform: a specific case

One thing that cannot be established with the general framework that was used in the previous section is the contributions that the four different instruments make in restoring fiscal sustainability. In order to be able to say something about that, we respecify our model such that it produces analytical solutions. In particular, we choose preferences of the isoelastic type with coefficients of relative risk aversion equal to 1/2:

$$
\begin{aligned}
u_t = {} & 2\left((c^y_t - 1/2\beta l_t^2)^{1/2} + \frac{\zeta_{t+1}}{1+r}(c^o_{t+1})^{1/2}\right) \\
& + 2\alpha^r\left((b^r_t)^{1/2} + \frac{\zeta_{t+1}}{1+r}(b^r_{t+1})^{1/2}\right) \\
& + 2\alpha^u\left((b^u_t)^{1/2} + \frac{\zeta_{t+1}}{1+r}(b^u_{t+1})^{1/2}\right)
\end{aligned}
\tag{6.1}
$$

Here, we use $\alpha^r > 0$ and $\alpha^u > 0$ to express the preference for rival and pure public goods (relative to private consumption goods).

The equivalents of optimality conditions (5.1), (5.2), (5.3) and (5.6) are now as follows:

$$
b^r(\mathrm{CP}) = \frac{(\alpha^r)^2 w^2 (1-2\tau(\mathrm{CP}))^2}{2\beta} \tag{6.2}
$$

$$
p(\mathrm{CP}) = \frac{1}{(\alpha^r)^2} b^r(\mathrm{CP}) \tag{6.3}
$$

$$
b^u_t(\mathrm{CP}) = (\frac{\alpha^u}{\alpha^r})^2 n_t^2 b^r(\mathrm{CP}) \qquad t = 1, 2+ \tag{6.4}
$$

$$
b^r(\mathrm{CP}) = \frac{N^y \frac{w^2}{\beta}\tau(\mathrm{CP})(1-\tau(\mathrm{CP})) - D_0}{N + N^u(\frac{\alpha^u}{\alpha^r})^2 + R(\frac{1}{\alpha^r})^2} \tag{6.5}
$$

Here, we use as shorthand notation R, which is defined as $(1+r)/r$ and N^u, which is defined as $n_1^2 + n_{2+}^2/r$.

Like in the previous section, combining the preference curve, which is now given by equation (6.2), and the budget constrant curve, which is now given by equation (6.5), yields the solution of the model. Due to the quadratic nature of the two curves, the solution for the optimal tax rate is given by a quadratic formula:

$$
\tau(\mathrm{CP}) = \frac{1}{2}\left[1 - \sqrt{\frac{N^y \frac{w^2}{\beta} - 4D_0}{N^y \frac{w^2}{\beta} + 2\frac{(\alpha^r w)^2}{\beta}\left(N + N^u(\frac{\alpha^u}{\alpha^r})^2 + R(\frac{1}{\alpha^r})^2\right)}}\right] \tag{6.6}
$$

The roots of this equation is only real if $N^y(w^2/\beta) - 4D_0 > 0$. We assume this is the case. Basically, this means that public debt should not be too large or the tax base should not be too small. In both cases, there would be no value for the tax rate that would render public finances sustainable (recall that tax revenues in the model are quadratic in the tax rate and thus cannot take any value). Furthermore, we assume the numerator of the fraction under the square root in equation (6.6) to be smaller than the denominator. This assumption is valid when D_0 is positive, zero or moderately negative. Under this assumption, the solution for the tax rate is strictly between zero and 1/2.

Combining equation (6.6) and equation (6.2), we can derive the solution for the optimal consumption of rival public goods:

$$b^r(\mathrm{CP}) = \frac{(\alpha^r w)^2}{2\beta}\left(\frac{N^y\frac{w^2}{\beta} - 4D_0}{N^y\frac{w^2}{\beta} + 2\frac{(\alpha^r w)^2}{\beta}\left(N + N^u(\frac{\alpha^u}{\alpha^r})^2 + R(\frac{1}{\alpha^r})^2\right)}\right) \tag{6.7}$$

Equation (6.6) shows nicely the effects of a longevity boost (an increase of N and N^u), a fertility bust (a decrease of N^y) and an increase of initial statutory debt (an increase of D_0): an increase in the tax rate $\tau(\mathrm{CP})$. Additionally, equation (6.7) show that the consumption of rival public goods decreases in all three cases. Both results recall the results of the previous section. If we invoke equation (6.3), we derive that also transfers will decrease in all three cases. If we invoke equation (6.4), we derive that in general also the consumption of pure public goods will decrease. Only in the case of a longevity boost, the effect upon consumption of the pure public good in period t=2+ is indeterminate, a result that was also derived in the previous section.

Figure 3 illustrates the effects of a longevity boost and of a fertility bust. In both cases, we have an increase of the sustainability gap. This does not affect the preference curve, but shifts downward the budget constraint curve. Hence, optimal policies move from point A to point B. This is a movement both to the right and downwards in the first quadrant of Figure 3. Hence, in all three cases there will be an increase of the tax rate, lower consumption of rival public goods, lower transfers and lower consumption of pure public goods. The only exception is the consumption of pure public goods in the future in case of a longevity boost. The expansion of the population that features this case increases the consumption of the pure public good in the future (to illustrate, the ppc curve changes to ppc' in Figure 3). Hence, the effect on future pure public goods consumption, b^u_{2+}, can have either sign. The extent to which the increase of the ageing problem translates into lower expenditure on rival public goods, pure public goods or transfers to the elderly, depends on preference parameters, α^r, α^u and on demographics, n^o_{2+} and n^y_{2+}.

[Figure 3 about here.]

How much do each of the four types of government policies contribute to restoring sustainability? To answer this question, we take the equation for the implicit government debt

(4.5). The demographic variables n_1, n_{2+}, n_1^y, n_{2+}^y, n_1^o and n_{2+}^o in this equation reflect the prospect of an ageing population. However, the policy variables b_1^r, b_{2+}^r, b_1^u, b_{2+}^u, p_1, p_{2+}, τ_1 and τ_{2+} in the equation refer to optimal policies based on the expectation of a stable population. Using the optimality conditions (6.2), (6.3) and (6.4), we write all policy variables in terms of $b^r(\text{IN})$, the initial value of b^r. This yields the following expression:

$$G_0 = b^r(\text{IN})\left(n_1 + \frac{n_{2+}}{r}\right) + b^r(\text{IN})\left(\frac{\alpha^u}{\alpha^r}\right)^2 n_1^2\left(\frac{1+r}{r}\right) + b^r(\text{IN})\left(\frac{1}{\alpha^r}\right)^2\left(n_1^o + \frac{n_{2+}^o}{r}\right) - \frac{w^2}{4\beta}\left(1 - \frac{2\beta b^r(\text{IN})}{(\alpha^r w)^2}\right)\left(n_1^y + \frac{n_{2+}^y}{r}\right) \tag{6.8}$$

Next, we do the same for optimal policies that are based on the information that the population will age in periode t=2+. This results in an expression in terms of $b^r(\text{CP})$:

$$H_0 = b^r(\text{CP})\left(n_1 + \frac{n_{2+}}{r}\right) + b^r(\text{CP})\left(\frac{\alpha^u}{\alpha^r}\right)^2\left(n_1^2 + \frac{n_{2+}^2}{r}\right) + b^r(\text{CP})\left(\frac{1}{\alpha^r}\right)^2\left(n_1^o + \frac{n_{2+}^o}{r}\right) - \frac{w^2}{4\beta}\left(1 - \frac{2\beta b^r(\text{CP})}{(\alpha^r w)^2}\right)\left(n_1^y + \frac{n_{2+}^y}{r}\right) \tag{6.9}$$

We now subtract H_0 from G_0 and divide the result by the term $b_1^r(\text{IN}) - b^r(\text{CP})$, which is irrelevant for the current purpose. This yields an expression with four elements. These four elements reflect the contribution to restoring sustainability made by the four types of government policies, *i.e.* rival public goods policies, pure public goods policies, transfer policies and tax policies respectively:

$$\frac{G_0 - H_0}{b^r(\text{IN}) - b^r(\text{CP})} = \left(n_1 + \frac{n_{2+}}{r}\right) + \left(\frac{\alpha^u}{\alpha^r}\right)^2\left(n_1^2 + \left[\frac{b^r(\text{IN})n_1^2 - b^r(\text{CP})n_{2+}^2}{b^r(\text{IN}) - b^r(\text{CP})}\right]\frac{1}{r}\right) + \left(\frac{1}{\alpha^r}\right)^2\left(n_1^o + \frac{n_{2+}^o}{r}\right) + \frac{1}{2(\alpha^r)^2}\left(n_1^y + \frac{n_{2+}^y}{r}\right) \tag{6.10}$$

Upon inspection, we derive that the role of cutting expenditure on rival public goods is larger, the higher is α^r. A higher value of α^r indicates a stronger preference of households for this type of consumption goods. As an implication, this type of consumption should contribute relatively strongly to restoring sustainability. In a similar vein, the role of a change in the consumption of pure public goods is larger when α^u is higher.

The intensity of the ageing process plays a role as well. The more the population will age, the larger will be the role of transfer policies and the smaller that of tax policies. The reason is that the older the population becomes, the more effective will be transfer policies and the less effective tax policies. Whether ageing takes the form of a longevity boost or a fertility bust is not relevant here.

Finally, population size is relevant for the contribution that is required from pure public goods policies. An expansion of the population increases the benefit/cost ratio of pure public goods, as we have noticed above. Hence, in case of an expanding population, the contribution required from pure public goods policies is relatively small. It can be as low as zero or even negative. In case of a shrinking population, the opposite occurs: the contribution made by pure public goods policies is then relatively large.

7 The impact on the generational balance

Above, we have established that the perspective of population ageing calls for immediate action and we have focussed on the contributions of the four types of policies that we have distinguished. What does this imply for the balance between generations? This question will be picked up now. We analyse the generational implications for each of the four policies in turn.

Let us start with tax policies. As shown, optimal tax policies feature smoothing of the tax rate over time. Given our assumptions on population structure and labour productivity, the same applies to tax revenues. Consequently, all generations will be hurt to the same extent by optimal tax policies, except for the currently old. As they have already retired from the labour market, they escape the higher tax rate.

The case of optimal transfer policies is different. Although optimal transfer policies exhibit smoothing as well, they do not treat the currently old differently from other generations. Indeed, under optimal transfer policies, there is perfect balance between all generations involved.

The case of optimal rival public goods policies is in between the two previous cases. Like optimal tax policies and optimal transfer policies, optimal rival public goods policies obey the smoothing principle. Optimal public goods policies do not exclude the currently old; still, the effect on the currently old is smaller than that on other generations as the currently old face lower consumption in one stage of their life cycle only.

The case of pure public goods policies is different from the other types of policies. Optimal pure public goods policies may imply both an increasing or a decreasing time profile of consumption. Hence, current generations, young and old, will suffer more or less from a change in pure public goods consumption than future generations.

8 The implied debt accumulation paths

What do we know about the dynamics of the public debt? From period t=2 onwards, the public debt is stable, or

$$\Delta D_{2+} \equiv D_{2+} - D_1 = 0 \tag{8.1}$$

This can be formally derived by leading the intertemporal government budget constraint in equation (4.5) one period and two periods, resulting in expressions for D_1 and D_{2+}, and then subtracting the equation for D_1 from that for D_{2+}. This stability of the public debt reflects the stability of the population from period t=2 onwards.

We can apply the same procedure to find an expression for $\Delta D_1 = D_1 - D_0$:

$$\Delta D_1 = -\Delta\left(n_{2+}b_{2+}^r + b_{2+}^u + n_{2+}^o p_{2+} - \frac{w^2}{\beta}n_{2+}^y \tau_{2+}(1-\tau_{2+})\right) \tag{8.2}$$

This expression tells us two things. First, the change in the public debt in period t=1 equals minus the change in the primary deficit from t=1 to t=2+, or the change in the primary balance from t=1 to t=2+. This tells us that in general the prospect of future population ageing, via an increase in the primary deficit, calls for a decumulation of the public debt. Second, by how much the debt will be decumulated, is unknown a priori. It depends among other things on the policies that are used to restore fiscal sustainability. The latter can be illustrated by elaborating equation (8.2) for the four types of policies.

That is what we will do now. We treat the four types of policies in isolation. That is, we change only the policy instrument in question to ensure fiscal sustainability and hold the other three policy instruments fixed at their initial levels.

For optimal tax policies, indicated with index TX, we have the following condition:

$$\Delta D_1(\mathrm{TX}) = -b^r(\mathrm{IN})\Delta n_{2+} - p(\mathrm{IN})\Delta n_{2+}^o + \frac{w^2}{\beta}\tau(\mathrm{TX})(1-\tau(\mathrm{TX}))\Delta n_{2+}^y \tag{8.3}$$

Similar expressions apply to the cases of optimal transfer policies (index TR), optimal rival public goods policies (index RP) and optimal pure public goods policies (PP):

$$\Delta D_1(\mathrm{TR}) = -b^r(\mathrm{IN})\Delta n_{2+} - p(\mathrm{TR})\Delta n_{2+}^o + \frac{w^2}{\beta}\tau(\mathrm{IN})(1-\tau(\mathrm{IN}))\Delta n_{2+}^y \tag{8.4}$$

$$\Delta D_1(\mathrm{RP}) = -b^r(\mathrm{RP})\Delta n_{2+} - p(\mathrm{IN})\Delta n_{2+}^o + \frac{w^2}{\beta}\tau(\mathrm{IN})(1-\tau(\mathrm{IN}))\Delta n_{2+}^y \tag{8.5}$$

$$\begin{aligned}\Delta D_1(\mathrm{PP}) = &-b^r(\mathrm{IN})\Delta n_{2+} - \Delta b_{2+}^u(\mathrm{PP}) - p(\mathrm{IN})\Delta n_{2+}^o \\ &+ \frac{w^2}{\beta}\tau(\mathrm{IN})(1-\tau(\mathrm{IN}))\Delta n_{2+}^y\end{aligned} \tag{8.6}$$

We will consider the longevity boost scenario and the fertility bust scenario in turn. We start with the case of an increase in life expectancy.

8.1 The longevity boost scenario

If the government adopts tax policies to curb the fiscal effects of a longevity boost, the public debt will decrease. This reflects the increase in the primary deficit one period later: spending on rival public goods and on transfers will increase (in line with the expansion of the elderly population), tax revenues will remain constant (the youngest cohort will remain constant) and spending on pure public goods will not change (initial policies are based on the expectation of a stable population).

Equation (8.4) tells us that the public debt will also decrease if transfers are reformed to restore sustainability. But, as $p(\mathrm{TR}) < p(\mathrm{IN})$, debt decumulation under optimal transfer policies will be smaller than under optimal tax policies. Correspondingly, the public debt will stabilize at a higher level. The reason is that as transfers to the elderly are reduced, the primary deficit will increase less, as compared to the case of optimal tax policies. Intuitively, the instrument of transfer policies becomes more effective over time as the number of elderly increases, so that the period t=1 debt reduction can be lower.

Also when rival public goods policies are used to restore sustainability, the public debt will decrease (equation (8.5)). As $b^r(\mathrm{RP}) < b^r(\mathrm{IN})$, debt decumulation under optimal rival public goods policies will be smaller than under optimal tax policies; it will be faster than the debt decumulation under optimal transfer policies, however. The reason is that expenditure on rival public goods increases proportionally with the population and thus less than proportionally with the elderly population, whereas transfer expenditure increases proportionally with the elderly population. Appendix A contains a formal proof of this proposition.

Optimal pure public goods policies stand out as they do not feature smoothing. This explains the additional term in equation (8.6). Recall from the previous section that, due to the projected expansion of the population in the longevity boost scenario, $b_2^u(\mathrm{PP}) > b_1^u(\mathrm{PP})$. Hence, debt decumulation will be larger than in all three other cases. The reason is that primary government expenditure now increases not only on account of an increasing number of consumers, but also on account of higher spending per consumer.

Wrapping up, the four types of government policies differ in the size of debt reduction. They have in common that the public debt should be decreased in period t=1 in anticipation of the ageing of the population from t=1 to t=2+. Is this equally true in the case of a fertility bust?

8.2 The fertility bust scenario

In the case of a fertility bust, optimal tax policies will imply a decrease in the public debt in period t=1, if two conditions are met: tax revenues are increasing in the tax rate (the tax rate is on the lefthand side of the Laffer curve) and spending on rival public consumption goods is lower than tax revenues. The first condition is a feature of optimal policies, the second condition an empirical one that will in most countries be met since spending on rival consumption goods is only one of the reasons for taxation.

Under optimal transfer policies, the public debt will also be decreased, but to a smaller extent than under optimal tax policies (equation (8.4)). Because of the decrease in the number of youngsters, the effectiveness of tax increases diminishes over time, whereas the effectiveness of transfer policies remains unchanged. In turn, debt decumulation under optimal rival public goods policies will be faster than under optimal transfer policies (8.5). Due to the decrease in the number of young persons, economizing on the spending on rival public goods becomes less effective over time than economizing on transfers to the elderly.

Equation (8.6) gives the fiscal effect of optimal pure public goods policies. In the case of a fertility bust, it will be optimal to reduce the spending on pure public goods from period t=1 to period t=2+. Hence, compared to the case of optimal transfer policies, the debt reduction in period t=1 should be smaller. The lower spending on public goods and the lower primary deficit that results from it, require the government to lower the amount of debt reduction. We cannot even exclude that the effect will be negative, which comes down to a debt increase.

Wrapping up, the fertility bust scenario differs from the longevity boost scenario only in the case of pure public goods policies. Unlike the case of a longevity boost, the case of a fertility bust may feature an increase of the public debt. The reason is that the need for public debt reduction will be smaller as the fertility bust and the concomitant decline of the population reduce the future spending on pure public goods.

We can study the change in public debt also for the case of combined optimal policies. Given the results for the four types of policies, the case of combined optimal policies is rather obvious. Formally, the change in public debt in periods t=2+ and t=1 is as follows:

$$\Delta D_2(\mathrm{CP}) = 0 \tag{8.7}$$

$$\Delta D_1(\mathrm{CP}) = -b^r(\mathrm{CP})\Delta n_{2+} - p(\mathrm{CP})\Delta n^o_{2+} - \Delta b^u_{2+}(\mathrm{CP}) + \frac{w^2}{\beta}\tau(\mathrm{CP})(1-\tau(\mathrm{CP}))\Delta n^y_{2+} \tag{8.8}$$

In case of a longevity boost, the first three terms on the RHS of equation (8.8) are negative, whereas the fourth term is zero. This unambiguously implies debt decumulation. In case of a fertility bust, the first and third term are positive, the second term is zero and the fourth term is negative. A fertility bust thus also calls for debt decumulation, but only if the spending on rival and pure public goods is sufficiently low.

9 Concluding comments

Most of our conclusions stem from a more general model, some of them rely on more specific assumptions regarding preferences and technology. In all cases, two assumptions are made of which the relevance needs further discussion. One is that of imperfect capital markets, the other one that of a small open economy.

Throughout the paper, optimal policies feature a positive labour income tax rate. This is due to our assumption of lacking private capital markets. Given that households cannot use private capital markets to smooth their consumption over the life-cycle, tax policies are beneficial in this respect. Should we have assumed perfect capital markets, the optimal labour income tax rate would have been zero. Taxes would no longer have the benefit of smoothing consumption over the life-cycle, but would remain costly by distorting the households labour-leisure decision. The government would then instead rely on negative transfers to the elderly in order to collect revenues.

A zero labor income tax would change some of our results: the contribution made by tax policies to fiscal sustainability would obviously be zero and the imbalance between the burdens imposed on the currently old and the other generations would for this reason be smaller. The rest of our conclusions would remain unchanged however.

Our analysis explores the case of a small open economy. Although most of the policies that are considered to cope with the ageing crisis belong to the national domain, the case of a closed economy is more general and therefore interesting in itself. Further research is needed to see how the endogeneity of factor prices that a move from the concept of a small open economy to that of a closed economy would bring about, will change our results.

In order to assess the quantitative aspects of policy interventions, our approach needs to be supplemented. Indeed, models that include many more generations and a more detailed modelling of policy institutions allow a better assessment of the numerical aspects of policy reform. Models that distinguish households also with respect to educational class allow for an analysis of the intragenerational effects of policy reforms. Policymakers may want to have such detailed information before choosing between different policy alternatives.

Acknowledgements

Thanks to Bas Jacobs and Thijs Knaap for comments on an earlier version of this paper, to Antonio Afonso who discussed a previous version of the paper at the 2008 Annual IIPF Conference and to Richard Rosenbrand for graphical assistance. Next, thanks for the comments made by two anonymous referees from this journal and by Alessandro Cigno, editor of this journal. The ideas in this paper are strictly personal and do not necessarily correspond to those of UvA, CPB, or Netspar.

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Appendix A

This appendix proves that debt decumulation will be faster under optimal rival public goods policies than under optimal transfer policies in case of a longevity boost. From equations (8.4) and (8.5), we can derive that this is equivalent with the condition $b^r(\mathrm{RP}) + p(\mathrm{IN}) > b^r(\mathrm{IN}) + p(\mathrm{TR})$, so our proof will be that this inequality holds true.

The first step is to apply the intertemporal government budget contraint, (4.5), to the case of optimal transfer policies:

$$p(\mathrm{TR})N^o = -D_0 - b^r(\mathrm{IN})N - \left(b_1^u(\mathrm{IN}) + \frac{b_{2+}^u(\mathrm{IN})}{r}\right) + \tau(\mathrm{IN})(1-\tau(\mathrm{IN}))\frac{w^2}{\beta}N^y$$

Do the same for optimal rival public consumption policies:

$$b^r(\mathrm{RP})N = -D_0 - \left(b_1^u(\mathrm{IN}) + \frac{b_{2+}^u(\mathrm{IN})}{r}\right) - p(\mathrm{IN})N^o + \tau(\mathrm{IN})(1-\tau(\mathrm{IN}))\frac{w^2}{\beta}N^y$$

Combining these two equations, we derive the following equality:

$$p(\mathrm{TR})N^o + b^r(\mathrm{IN})N = b^r(\mathrm{RP})N + p(\mathrm{IN})N^o$$

We use this equality condition to find an expression for $b^r(\mathrm{IN}) + p(\mathrm{TR})$ in terms of $b^r(\mathrm{RP}) + p(\mathrm{IN})$:

$$\begin{aligned}
b^r(\mathrm{IN}) + p(\mathrm{TR}) &= \Big(Nb^r(\mathrm{IN}) + Np(\mathrm{TR})\Big)/N = \Big((Nb^r(\mathrm{IN}) + N^o p(\mathrm{TR})) + (N-N^o)p(\mathrm{TR})\Big)/N \\
&= \Big((Nb^r(\mathrm{RP}) + N^o p(\mathrm{IN})) + (N-N^o)p(\mathrm{TR})\Big)/N \\
&= \Big((Nb^r(\mathrm{RP}) + Np(\mathrm{IN})) - (N-N^o)p(\mathrm{IN}) + (N-N^o)p(\mathrm{TR})\Big)/N \\
&= (b^r(\mathrm{RP}) + p(\mathrm{IN})) - (N^y/N)(p(\mathrm{IN}) - p(\mathrm{TR}))
\end{aligned}$$

Recalling that $p(\mathrm{TR}) < p(\mathrm{IN})$, we derive that $b^r(\mathrm{RP}) + p(\mathrm{IN})$ is larger than $b^r(\mathrm{IN}) + p(\mathrm{TR})$ $\square$

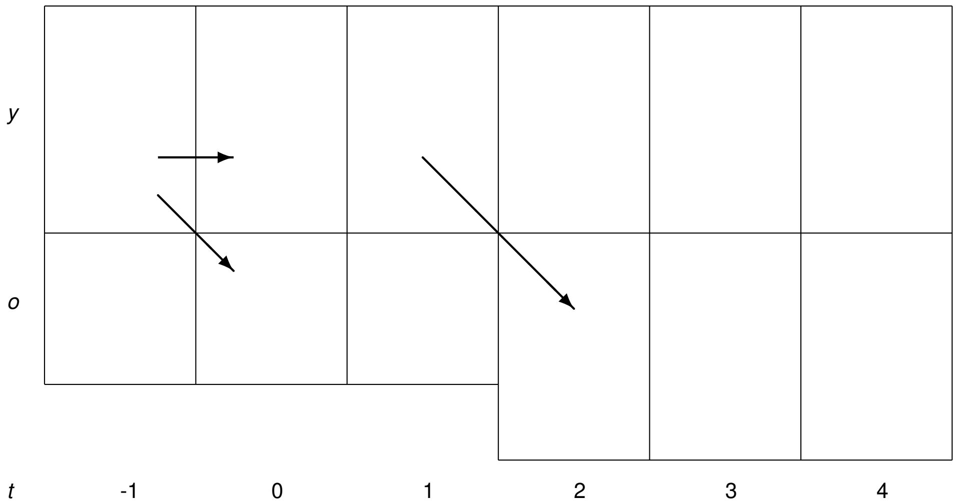

Figure 1 Longevity boost

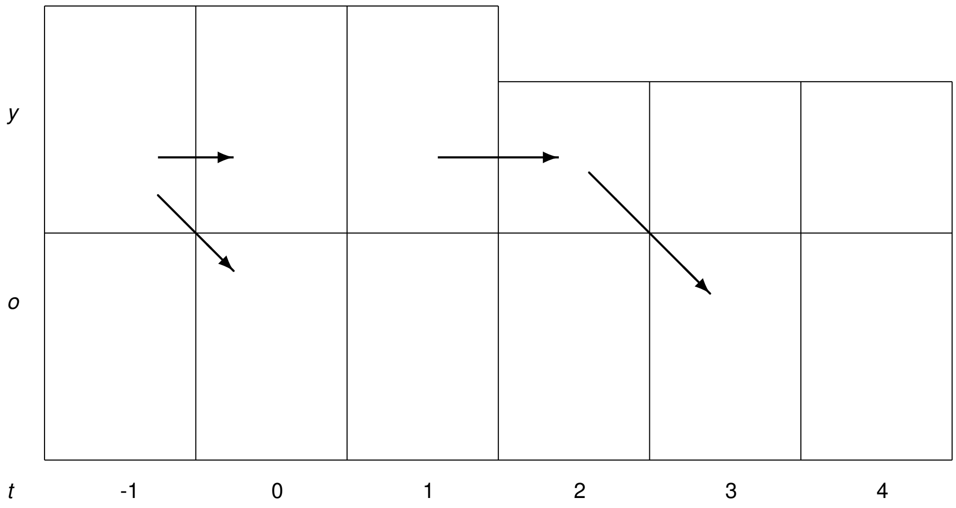

Figure 2 Fertility bust

Figure 3 The optimal combination of policies