

# The hidden treasure of collective defined benefit pension schemes – the good thing about front-loading of pension contributions

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## **Abstract**

Collective defined benefit (DB) pension schemes typically feature front-loaded pension contributions (or back-loaded pension accruals). This front-loading has two effects: it makes pension schemes actuarially unfair and hinders labour market efficiency. This paper shows that front-loading may also have a good side. In particular, if the labour supply elasticity of old workers is large relative to that of young workers, front-loading of contributions may increase aggregate labour supply. In case aggregate labour supply is sub-optimally low due to, *e.g.*, the taxation of labour income, this dampens the welfare loss due to front-loading. Front-loading may even imply higher welfare than actuarially fair contribution policies.

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## 1. Introduction

Worldwide, pension reform is at the top of the policy agendas. The Netherlands is no exception. The last decade has witnessed several reforms in this country, among which the move from final pay to average pay, the shift from defined benefit (DB) towards collective defined contribution (DC) and the increase of the retirement age. Moreover, supervision of pension schemes became more strict as the government decided that market-based interest rate rather than a fixed interest rate should be used to assess the funding ratios of pension funds.

As other countries, the Netherlands abolished the financial incentives in early retirement schemes to retire long before the official retirement age. As a result, the labour market participation of older workers increased dramatically. Discussion about pension reform has not ended yet, however. The last few years, the system of frontloading pension contributions has come under severe attack. In July this year the government announced to start abolishing the scheme in 2020.

Frontloading is inherent to collective defined benefit (DB) pension schemes since these feature age-independent contribution and accrual rates. As pension contributions increase in value over time, this means that pension contributions are front-loaded relative to pension accruals (or, equivalently, pension benefits are back-loaded). This front-loading has two effects. First, it renders pension schemes actuarially unfair. In particular, young workers pay more than they earn in terms of pension accruals, whereas old workers earn more than they contribute to the pension scheme. If workers had about equally long labour market careers and there was little mobility between jobs, the front-loading would have little relevance. With increasing labour market flexibility and mobility, the practice of front-loading is becoming more important, however.

Second, the front-loading of pension contributions may hinder labour market efficiency. As both pension contributions and accruals are linked to labour income, the difference can be regarded an implicit labour income tax (or subsidy, if negative). This may distort labour supply decisions, inducing young workers to supply less labour and old workers to supply more labour than is in their interest. Moreover, the front-loading of contributions may hamper labour mobility in and out of the labour market and between jobs. Indeed, it makes it unattractive for a worker to withdraw from the labour market before the retirement date as this implies that he or she will forego future implicit subsidies. The same holds true with respect to moving to another job that features a DC pension scheme or no pension scheme at all.

This paper shows that front-loading of pension contributions may also have a good side. In particular, if the labour supply elasticity of older workers is large relative to that of younger workers, front-loading of contributions may add to aggregate labour supply. In itself, this is not a source of a welfare gain. Indeed, both the reduction of labour supply by young workers and the (larger) increase of it by old workers contribute to the welfare loss of front-loaded contributions. But if aggregate labour supply is initially sub-optimally low, e.g. due to a tax on labour income, an increase in aggregate labour supply may be beneficial. It expands the tax base, which allows the government to reduce the tax rate and thereby improve welfare.

The latter effect upon welfare may even dominate the former effect, in which case the front-loading of pension contributions is welfare-increasing.

This paper is structured as follows. Section 2 demonstrates that DB pensions feature front-loading of pension contributions relative to pension accruals. Section 3 discusses some of the economic effects of this front-loading. Section 4 then sets up a stylized two-period model in order to explore the labour market and welfare implications of front-loaded pension contributions. It shows that if labour supply elasticities are positive, the front-loading of pension contributions will be unambiguously welfare-reducing. Section 5 extends the model with a government financing requirement that is met by taxing labour income. This section shows that the extension with public consumption reinforces the earlier results if young and old workers have common labour supply elasticities. But if the labour supply elasticity of old workers exceeds that of young workers, the welfare loss is reduced, due to the expansion of the tax base and the reduction of the rate of general taxation that stems from it. The front-loading principle may even produce higher welfare than an actuarially fair financing scheme. Section 6 explores numerically the general case of a discrete policy reform. In general, the welfare effect can have either sign, reflecting the battle between the introduction of a new distortion and the alleviation of an already existing distortion. Finally, section 7 offers some concluding remarks.

## **2. The implicit taxes and subsidies implied by front-loaded pension contributions**

In general, collective DB schemes treat participants of different age uniformly. Both pension contribution rates and accrual rates are uniform, *i.e.* do not differentiate with respect to age. This type of uniformity is intrinsically linked with the collective nature of the pension scheme. In many cases, this uniformity is also legally required.

In general, the uniformity is actuarially unfair. A euro of pension contributions made by a young worker has greater economic value than a euro contributed by an old worker. The former euro has a longer time to accumulate and earn interest than the latter one. Note that the accrual of pension benefits that is achieved by contributing a euro is also larger for the young than for the old worker if we assume indexation of benefits to productivity growth. However, if the rate of capital market return is larger than the rate of productivity growth – which in general will be the case, pension contributions will be more front-loaded than pension accruals. Hence, the young worker contributes too much and the old worker too little relative to what they receive in return in the form of future pension benefits.

Since both contributions and accruals are related to individual labour income, the differences between the two can be interpreted as implicit taxes (in case of young workers) and implicit subsidies (in case of old workers). Demographic and economic factors determine how large are these taxes and subsidies. A crucial demographic factor is the length of the working phase during which the worker pays contributions and accumulates pension rights. Crucial economic factors are the capital market rate of return,  $r$ , and the rate of labour productivity growth,  $g$ . Actually, it is the difference between the two that determines how large are the implicit taxes and subsidies. In the polar case  $r = g$ , these taxes and subsidies vanish, but for any positive difference ( $r > g$ ) the implicit pension taxes and subsidies will be

non-zero, except for a well-defined middle-aged worker. In general, the larger the gap between  $r$  and  $g$ , the bigger the taxes and subsidies.

Let us establish this formally. We decompose the life cycle of an individual in two parts: the working or pension contribution phase and the retirement phase. Let us use  $T_R$  to denote the length of the working phase and  $T_L$  to denote the length of life.  $T_L - T_R$  is then the length of the retirement phase. We model a general DB scheme. This scheme encompasses the final pay scheme, in which pension benefits are linked to last-earned wages, and the average pay scheme, in which the average of labour income over a worker's career determines the pension benefits. For simplicity, we assume full indexation of pensions to labour income growth. However, this assumption is by no means crucial: the cases of partial indexation, price indexation or lack of indexation (nominal pensions) are natural extensions of our model.

Importantly, for ease of exposition, we model the pension scheme as an individual scheme, *i.e.* there are no (contingent or non-contingent) transfers between generations. Modelling the pension scheme as a collective scheme with many generations and (also many) intergenerational transfers would help to stay closer to real-world pension schemes. But it would also take considerably more space, make the analysis more complex and distract from our basic message in this section, which is that the principle of uniform contribution rates and uniform build-up rates implies that pension contributions are front-loaded relative to the accumulation of pension rights.

As to  $r$  and  $g$ , we assume they are positive, non-stochastic and constant through time. Furthermore, we will assume  $r > g$  throughout this section. This is a quite weak assumption. Empirically, the capital market rate of return  $r$  generally exceeds the rate of economic growth (Abel *et al.* (1989), Piketty (2014)). Moreover, population growth is generally positive, rendering the rate of economic growth higher than the rate of productivity growth  $g$ .

We start to define the amount of pension wealth that the worker needs to have accumulated upon retirement in order to finance wage-indexed pensions that we take to be a fraction  $\beta$  of contemporaneous labour income (throughout this section, all flow and stock variables will be defined on a primo basis, *i.e.* at the beginning of the year),

$$B_{T_R} = \sum_{t=T_R+1}^{T_L} \beta y_{T_R} \left( \frac{1+g}{1+r} \right)^{t-T_R} \quad (1)$$

where  $y_{T_R}$  denotes labour income at the time of retirement. We attach index  $T_R$  to the wealth variable  $B$  to make clear that wealth is measured in euros at the time of retirement of the worker.

If contributions to the pension scheme are proportional to labour income during the working career, we can express the total of pension contributions that an individual has made over his life-cycle at the time of retirement as follows,

$$\Pi_{T_R} = \sum_{t=1}^{T_R} \pi y_t (1+r)^{T_R-t} = \sum_{t=1}^{T_R} \pi y_{T_R} \left(\frac{1+r}{1+g}\right)^{T_R-t} \quad (2)$$

where  $\pi$  denotes the (age-independent) contribution rate. The most RHS expression is useful as it expresses  $\Pi_{T_R}$  as proportional with  $y_{T_R}$ .

As the scheme is individual,  $B_{T_R}$  and  $\Pi_{T_R}$  must be equal. We can use this information to derive the following expression for the contribution rate:

$$\pi = \sum_{t=T_R+1}^{T_L} \beta \left(\frac{1+r}{1+g}\right)^{T_R-t} / \sum_{t=1}^{T_R} \left(\frac{1+r}{1+g}\right)^{T_R-t} \quad (3)$$

Similarly, if the accrual of pensions is proportional to concomitant labour income, we can derive the following expression for the total of pension accruals at the time of retirement,

$$A_{T_R} = \sum_{t=1}^{T_R} \alpha y_t (1+g)^{T_R-t} = \sum_{t=1}^{T_R} \alpha y_{T_R} \quad (4)$$

where  $\alpha$  denotes the accrual rate. Again, the most RHS expression expresses  $A_{T_R}$  as proportional with  $y_{T_R}$ . Note that equation (4) reflects the assumption of full wage indexation of pension benefits, which renders the principles of final pay and average pay equivalent.

In order to calculate the accrual rate that ensures that the worker will receive the benefits that he has accumulated, we equate  $A_{T_R}$  to  $B_{T_R}$ . This gives the following expression for the accrual rate:

$$\alpha = \sum_{t=T_R+1}^{T_L} \beta \left(\frac{1+r}{1+g}\right)^{T_R-t} / T_R \quad (5)$$

In order to derive that pension contributions are decreasing with age, it suffices to take a look at the expression for the pension contribution in euros dated at time  $T_R$  at age  $\tau$  ( $\tau=1, \dots, T_R$ ), which reads as  $\pi y_\tau (1+r)^{T_R-\tau}$ . In any given year during the working phase, pension contributions are a factor  $1+g$  larger than those of the previous year due to growth of the contribution base, but a factor  $1+r$  smaller due to the shorter holding period. Pension contributions thus decline with rate  $(1+r)/(1+g)$  over the worker's life cycle. Pension accruals, on the other hand, are independent of age. To establish this, we write down the expression for the pension accrual, again in euros dated at time  $T_R$  at age  $\tau$  ( $\tau=1, \dots, T_R$ ), which reads as  $\alpha y_\tau (1+g)^{T_R-\tau}$ . Although in any year the pension accrual is a factor  $1+g$  higher than the previous year's accrual, full indexation of pension rights to labour productivity growth implies that the previous year's accrual grows at the same pace. The two mechanisms

thus cancel against each other. Hence, with pension contributions declining and pension accruals constant during the working phase, we conclude that pension contributions are more front-loaded than pension accruals.

Alternatively, we can analyse the life-cycle development of pension contributions and pension accruals in terms of contemporaneous labour income, expressed as  $\pi(1+r)^{T_R-\tau}$  and  $\alpha(1+g)^{T_R-\tau}$  respectively. Defined this way, pension contributions decline with factor  $1+r$  and pension accruals with factor  $1+g$  during the working phase. This alternative approach, obviously, yields the same conclusion: pension contributions are front-loaded as compared with pension accruals.

Combining this result with the fact that total pension contributions,  $\Pi_{T_R}$ , are equal to total pension accruals,  $A_{T_R}$ , it follows that pension contributions exceed accruals at younger ages and accruals are larger than contributions at older ages. To establish this formally, define net benefits from the pension scheme,  $Z_{T_R}$ , as accumulated pension rights minus pension contributions,  $A_{T_R} - \Pi_{T_R}$ , and differentiate it with respect to labour income at age  $\tau$  ( $\tau=1, \dots, T_R$ ). This gives  $\partial Z_{T_R} / \partial y_\tau = \alpha(1+g)^{T_R-\tau} - \pi(1+r)^{T_R-\tau}$ . Upon substitution of the derived expressions for  $\alpha$  and  $\pi$  in equations (3) and (5), this can be elaborated into the following:

$$\frac{\partial Z_{T_R}}{\partial y_\tau} = \frac{\alpha(1+g)^{T_R-\tau}}{\sum_{t=1}^{T_R} \frac{(1+r)^{T_R-t}}{(1+g)^{T_R-t}}} \left\{ \sum_{t=1}^{T_R} \frac{(1+r)^{T_R-t}}{(1+g)^{T_R-t}} - T_R \frac{(1+r)^{T_R-\tau}}{(1+g)^{T_R-\tau}} \right\} \quad (6)$$

The sign of the expression between accolades and thus of  $\partial Z_{T_R} / \partial y_\tau$  is negative (positive) if the worker is younger (older) than the critical age  $\tau^*$ ;  $\partial Z_{T_R} / \partial y_\tau$  is zero if the worker is  $\tau^*$  years old.  $\tau^*$  can be found by calculating the value of  $\tau$  that gives the expression between accolades the value of zero.

Let us visualize this. Figures 1 and 2 show the results of a calculation, based on an interest rate of 2 percent, a growth rate of labour productivity of 1 percent, a working phase of 45 years (age 20 to 64), a retirement phase of 20 years (age 65 to 84) and a replacement rate,  $\beta$ , of 100 percent.

Figure 1 shows the calculated development of pension contributions and accruals. Here, pension contributions are defined as  $\pi(1+r)^{T_R-\tau}$  and accruals as  $\alpha(1+g)^{T_R-\tau}$ , as in the above formula for  $\partial Z_{T_R} / \partial y_\tau$ . Pension contributions are seen to be front-loaded relative to pension accruals. For young workers, contributions exceed accruals, for old workers the reverse holds true. The two are about equal in the middle of the working phase. I don't want to emphasize any quantitative numbers. This conflicts with the stylized nature of the calculation. The figure only suggests that the differences between contributions and accruals can be large, especially at the beginning and towards the end of the working phase.

Figure 1: Pension contributions and accruals over the working phase of the life cycle

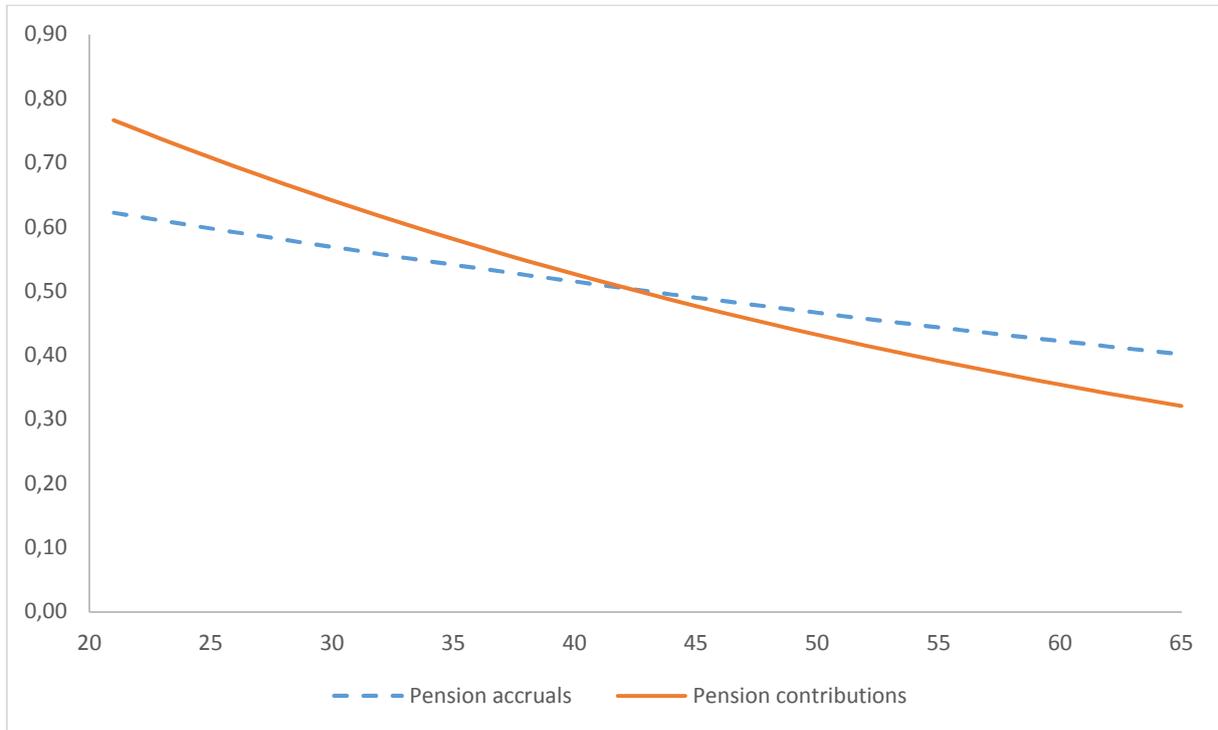


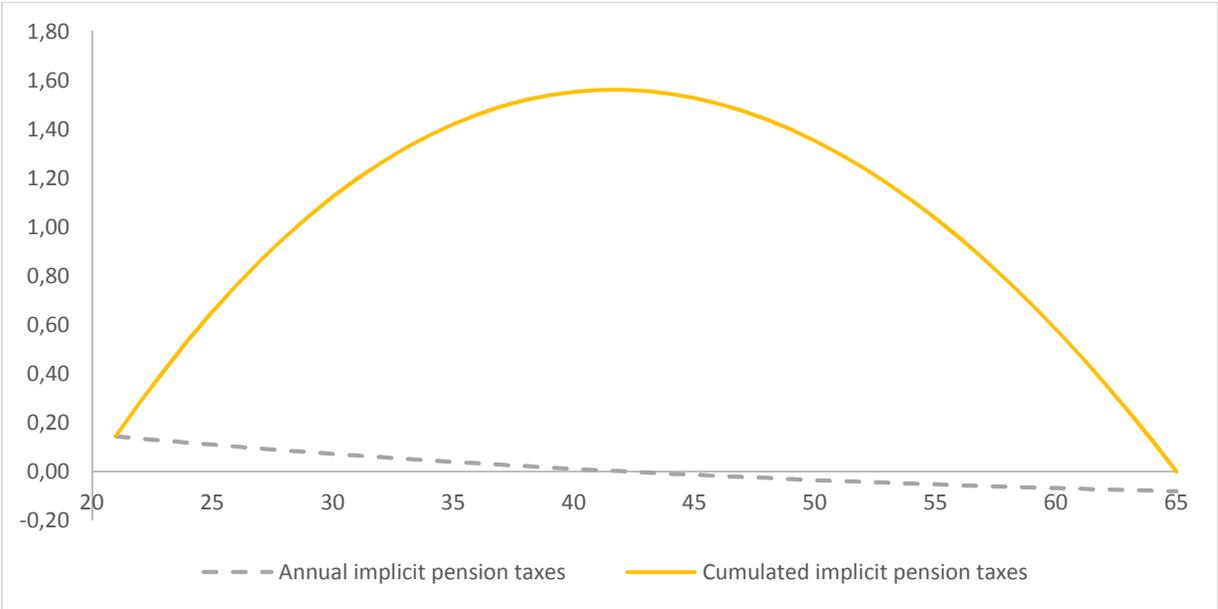
Figure 2 shows what this implies for implicit pension taxes and subsidies.  $\partial Z_{T_R} / \partial y_t$ , the annual implicit pension tax at age  $t$ , is displayed in Figure 2, together with its cumulative counterpart  $(\sum_{t=1}^{\tau} (\partial Z_{T_R} / \partial y_t) y_t)$ . As Figure 2 shows, the cumulative implicit pension tax is zero upon retirement: the (present value of the) implicit taxes paid at younger ages cancels exactly against the (present value of the) implicit subsidies received at older ages. The proof that  $\sum_{t=1}^{\tau} (\partial Z_{T_R} / \partial y_t) y_t = 0$  for  $\tau = T_R$  is straightforward.

As Figure 2 shows, the implicit pension tax is positive for the youngest ages and negative for the oldest ages. The cumulative implicit pension tax accumulates until the middle of the working period and decline after that age, reaching the zero level upon retirement. The above comment on the stylized nature of the calculations applies here as well. Figure 2 thus only suggests that the cumulative implicit pension tax can be huge. This suggestion is in line with the calculations in CPB (2013). This report calculates the net benefit of participation in a DB pension scheme if a person works full time until he or she is 45 years old and then stops his or her participation in the pension scheme (for example, because the labour market status of the person changes into that of a self-employed person). The report compares this net benefit with the net benefit that the same person would enjoy if he or she would work full-time until the retirement age. The report calculates that the net benefit of participation in the pension

scheme by the former person may be on average about 35 percent of pension income lower than that of the person who does not leave the pension scheme before retirement.

Obviously, calculations for a real-world individual worker may be quite different from the ones presented here. Generally, both the rate of return on savings and the rate of labour productivity growth fluctuate over time. Similarly, the rate of individual labour productivity growth differs from the macroeconomic growth rate. The participation profile is seldom flat, as assumed in the calculations here. Moreover, in the real world uniform policies apply to groups of heterogeneous workers, so that individual workers typically face transfers to or from other persons over their life-cycle. The pattern of pension contributions relative to pension accruals as analysed in this section should therefore be interpreted as the common element in a large set of patterns that are different for different workers.

Figure 2: Annual and cumulative implicit pension taxes over the working phase of the life cycle



### 3. Labour market effects of front-loaded pension contribution policies

Figures 1 and 2 are useful to discuss the labour market effects of the uniformity of contribution and accruals rates. In particular, Figure 1 can be used to discuss the incentives for labour supply by young and old workers. Since the pension contributions and the accruals relate to individual labour income, one can say that young workers pay an implicit tax to the pension scheme and old workers receive an implicit subsidy. These taxes and subsidies imply an incentive for young workers to reduce their labour supply and for the old workers to increase theirs. Both effects reflect the substitution between work and leisure. Note that

income or wealth effects do not play a role here as pension taxes and subsidies cancel out over (the working phase of) the life cycle.

Although the labour supply effects of taxes and subsidies are qualitatively different, the corresponding welfare effects may be similar. Indeed, both taxes and subsidies reduce welfare by distorting labour supply decisions. This does not say that the (labour supply and welfare) effects of workers of different age have the same order of magnitude. The few studies that have estimated how labour supply elasticities vary with age all suggest that labour supply elasticities increase strongly with age.

For example, French (2005) estimates that the labour supply elasticity is 0.3 for a 40-year old worker, but 1.1 for a worker of 60 years old. For males, Fenge *et al.* (2006) finds that the labour supply elasticity rises with age: compensated elasticities are estimated at 0.010 for 20-39 cohorts and 0.215 for 40-59 age cohorts. In contrast, for females the estimates do not differ much with respect to age; the corresponding elasticities are 0.527 and 0.565 respectively. The figures reported in French and Jones (2012) are close to those in French (2005): the estimated labour supply elasticity is 0.17 for 40-year old workers and 1.17 for workers with age 60.

The reason for the difference in elasticities is not fully clear. It may be that older workers are simply more responsive to financial incentives or that older workers have built up a larger stock of pension wealth. Another reason may be that it is less important for older than for younger workers to work for precautionary reasons: build up a stock of wealth in order to protect against future negative earnings shocks. Both French and Jones (2012) and Keane and Rogerson (2015) stress the role of the participation decision. Changes in financial incentives may induce young workers to change their hours of work. Older workers, on the other hand, may also choose along the extensive margin: retire at an earlier or later date.

There is a large literature on the question whether financial incentives offered by pension schemes have any effect upon retirement behaviour. Samwick (1998) finds that the accrual rate of retirement wealth does affect the probability of retirement. Similarly, Chan and Stevens (2004), Asch *et al.* (2005), Coile and Gruber (2007) and Mastrobuoni (2009) conclude that financial incentives determine the retirement behaviour of older workers. Euwals *et al.* (2010) and Hanel (2010) find the same applies to (early) retirement behaviour in the Netherlands and Germany respectively.

It remains a question whether changes in financial incentives that pertain to the distant future are as important as changes that apply to the near future. The evidence on time discounting (Frederick *et al.*, 2002) strongly suggests this is not the case. We will return to this issue below.

Figure 2 shows that it is generally unattractive to quit the labour market at older ages. A worker who decides at some point in the working phase to leave the labour market, will forego the implicit subsidies to labour market participation at older ages. For a worker who decides to move from a full-time to a part-time job, the same holds true, be it to a lesser degree. And the same also holds true for a worker who moves from a job that features a DB pension to a different job featuring a DC pension or no pension. This indicates that the front-loading of pension contributions that characterizes the DB pension scheme may reduce the mobility in and out of the labour market and on the labour market.

On the basis of this one would expect that the Netherlands with its prominent role of collective DB schemes features small numbers of people who change their labour market status from wage-earner to self-employed before retirement. This conflicts with the data however. Indeed, the data show a strong increase in the number of self-employed persons during the last 25 years. But other factors may have played a role here. In this regard, Van Es and Van Vuuren (2010) find that the increase in the number of self-employed persons should mainly be attributed to fiscal policies which aimed to make entrepreneurship more attractive in the Netherlands.

#### 4. A three-period life-cycle model

Now that we have seen that the principle of frontloading implies implicit taxes and subsidies (of which in particular the latter may be relevant for labour supply), we now set up a life-cycle model in order to explore in more detail the labour market and welfare effects of front-loaded pension contributions. We view the principle of frontloading as an optimal tax problem. This avoids being distracted by the many interesting facets of DB pensions such as risk sharing and redistribution between generations that are not directly relevant for the problem at hand.

The most simple life-cycle model that enables an analysis of front-loaded pension contribution policies is a three-period model, non-stochastic and without heterogeneity within age groups. This model describes the labour supply behaviour of a consumer in three periods: when he is young and working, say between age 25 and 45, and when he is old and working, say between age 45 and 65. In the third period, say, between age 65 and 85, the consumer is retired. The consumer chooses his labour supply in the first two periods and consumption in the third period such as to maximize his lifetime utility. The front-loading of pension contributions enters this decision problem by changing the price of leisure in the first and second period; as discussed in the previous section, front-loading pension contribution policies do not affect lifetime wealth.

We describe the preferences of the consumer by the following intertemporal utility function,

$$U = u_1(v_1) + u_2(v_2) + c_3, \quad (7)$$

where  $v_i$  denotes leisure in period  $i = 1, 2$  and  $c_3$  denotes consumption when retired. The utility function features positive and decreasing marginal utility with respect to leisure consumption in both periods.

The worker needs to finance his consumption during retirement out of the labour income earned in the two working periods,

$$c_3 = w_1 l_1 + w_2 l_2 = w_1(1 - v_1) + w_2(1 - v_2) \quad (8)$$

where  $l_i$  denotes labour supply in period  $i = 1,2$  and  $w_i$  denotes the wage rate in period  $i = 1,2$ . In deriving the most right-hand side expression in equation (8), we have used the constraint that leisure and labour supply add up to available time (normalized at one). Implicitly, equation (8) fixes the real interest rate equal to zero. The interest rate does not play an essential role in the present analysis, so we leave it out for clarity.

The utility function and budget constraint combine into the following Lagrangian function:

$$\mathcal{L} \equiv \{u_1(v_1) + u_2(v_2) + c_3\} + \lambda\{w_1(1 - v_1) + w_2(1 - v_2) - c_3\} \quad (9)$$

Necessary conditions for an interior optimum are that the derivatives  $\partial\mathcal{L}/\partial v_1$ ,  $\partial\mathcal{L}/\partial v_2$  and  $\partial\mathcal{L}/\partial c$  are zero. Combining these three first-order conditions gives the demand functions for leisure in periods 1 and 2,

$$v_1 = u_1'^{-1}(w_1) \equiv x_1(w_1) \quad (10)$$

$$v_2 = u_2'^{-1}(w_2) \equiv x_2(w_2) \quad (11)$$

where we have introduced the functions  $x_i$   $i = 1,2$  for convenience. Throughout this and the next section, we will assume that in both periods leisure is strictly positive and below one.

We model front-loaded pension contribution policies as an implicit tax on labour income in period 1 and an implicit subsidy on labour income in period 2. Using  $t_1$  and  $s_2$  to denote the rates of taxation and subsidization in the first and second period respectively, we can express the wage rate in period 1 as a function of  $t_1$  and the wage rate in period 2 as a function of  $s_2$ ,

$$w_1 = q_1(1 - t_1) \quad (12)$$

$$w_2 = q_2(1 + s_2) \quad (13)$$

where we use  $q_i$   $i = 1,2$  to denote productivity at age  $i$ . Combining things, we can now express the worker's utility  $U$  as a function of the tax rate  $t_1$  and subsidy rate  $s_2$ :

$$\begin{aligned}
U &= u_1 \left( x_1(q_1(1 - t_1)) \right) + u_2 \left( x_2(q_2(1 + s_2)) \right) \\
&+ q_1(1 - t_1) \left( 1 - x_1(q_1(1 - t_1)) \right) + q_2(1 + s_2) \left( 1 - x_2(q_2(1 + s_2)) \right)
\end{aligned} \tag{14}$$

This section assumes that the government's only role is to organize pension contribution policies. Hence, the government budget constraint relates the revenues from period-1 taxation to the spending on period-2 subsidies,

$$\begin{aligned}
q_1 t_1 (1 - v_1) &= q_2 s_2 (1 - v_2) \rightarrow \\
q_1 t_1 \left( 1 - x_1(q_1(1 - t_1)) \right) &= q_2 s_2 \left( 1 - x_2(q_2(1 + s_2)) \right)
\end{aligned} \tag{15}$$

where equations (10) to (13) have been used to derive the second line of equation (15).

Equation (15) differs from the usual budget constraint, as the pension tax and subsidy are implicit. Note that we have derived in section 2 that the present value of the implicit taxes paid by young workers equals that of the implicit subsidies received by old workers. Equation (15) is the two-period equivalent of that condition.

Total differentiation of equation (15) gives us an expression for  $dt_1/ds_2$ ,

$$\frac{dt_1}{ds_2} = \frac{q_2 l_2 (1 + \tilde{s}_2 \eta_2)}{q_1 l_1 (1 - \tilde{t}_1 \eta_1)} \tag{16}$$

where  $\tilde{s}_2$  and  $\tilde{t}_1$  are short-hand notations for  $s_2/(1 + s_2)$  and  $t_1/(1 - t_1)$  respectively. Further, we have used  $\eta_i$   $i = 1, 2$  to denote the elasticity of labour supply in period  $i$  with respect to that period's wage rate,  $w_i$ :  $\eta_i \equiv -x'_i(w_i)/((1 - x_i(w_i))/w_i)$   $i = 1, 2$ . Henceforth, the two labour supply elasticities will be assumed strictly positive:  $\eta_i > 0$   $i = 1, 2$ . The numerator and the denominator of the fraction at the RHS of equation (16) have their own interpretation. They correspond to the derivative of tax revenues to the tax rate and the derivative of subsidy outlays to the subsidy rate respectively. We will assume that these derivatives are positive, *i.e.* the economy is on the left side of the respective Laffer curves. Agell and Persson (2001) provides some empirical evidence for this condition. Hence,  $dt_1/ds_2 > 0$ .

Now we are ready to assess the welfare properties of pension contribution policies. Note that, as the implicit tax and subsidy rate are related,  $dU/ds_2$  must be elaborated as

$\partial U/\partial s_2 + \partial U/\partial t_1(dt_1/ds_2)$ . Hence, upon using equation (8) and (10), we derive the following expression for  $dU/ds_2$ :

$$\frac{dU}{ds_2} = q_2 l_2 \left( 1 - \left( \frac{1 + \tilde{s}_2 \eta_2}{1 - \tilde{t}_1 \eta_1} \right) \right) \quad (17)$$

Equation (17) allows to derive two propositions.

**Proposition 1:**

If the only role of the government is to organize pension contribution policies, the introduction of a marginal degree of front-loading in pension contribution policies exerts no effect upon welfare.

**Proof:**

The welfare effect of a marginal reform, *i.e.* a marginal increase in  $s_2$  from  $s_2 = t_1 = 0$ , is zero, as can be seen by inspection of equation (17).

**Proposition 2:**

If the only role of the government is to organize pension contribution policies, the introduction of a discrete (*i.e.*, more than marginal) degree of front-loading in pension contributions is strictly welfare-decreasing.

**Proof:**

In order to validate proposition 2, note that a discrete policy reform can be viewed as a succession of reforms, in each of which  $s_2$  exhibits a very small increase. The welfare effect of the first reform, the marginal reform, is zero (see proposition 1). To evaluate the welfare effect of the increase in  $s_2$  in the second step, we rewrite  $dU/ds_2$  as follows:

$$\frac{dU}{ds_2} = q_2 l_2 (1 + \tilde{s}_2 \eta_2) \left\{ \left( \frac{1}{1 + \tilde{s}_2 \eta_2} \right) - \left( \frac{1}{1 - \tilde{t}_1 \eta_1} \right) \right\} \quad (18)$$

The increase in  $s_2$  in the second step starts from  $s_2, t_1 > 0$ . The first term between accolades is now smaller than one, whereas the second term is larger than one. The expression between accolades is thus negative. The welfare effect of the increase in  $s_2$  in the second step must then be negative as well. Subsequent steps are similar to this second step. Hence, we conclude

that the introduction of a discrete degree of front-loading of pension contributions is welfare-reducing.

The impact of front-loaded pension contribution policies on welfare is larger, the higher are the labour supply elasticities (see equation (17)). The reason lies in the relation between the tax and subsidy rate. For a given increase in the subsidy rate  $ds_2$  the change in the tax rate  $dt_1$  will be larger the higher is one of the two elasticities. A higher period-1 elasticity implies that the period-1 tax base will be more eroded if the period-1 tax rate increases so that a higher period-1 tax rate increase is needed to balance the government budget. A higher period-2 elasticity implies that the period-2 subsidy base will be more expanded if the period-2 subsidy rate increases so that, again, a higher period-1 tax rate increase is needed to balance the government budget.

For later purposes, it is useful to have a look at the implication of the policy reform for aggregate labour supply. It is straightforward given the above equations to derive the following expression for the labour supply effect:

$$\frac{d(l_1+l_2)}{ds_2} = \left\{ -\frac{\eta_1}{1-t_1} \left( \frac{q_2(1+\tilde{s}_2\eta_2)}{q_1(1-\tilde{t}_1\eta_1)} \right) + \frac{\eta_2}{1+s_2} \right\} l_2 \quad (19)$$

Interestingly, the labour supply effect does not generally vanish. Indeed, in case of a marginal reform, *i.e.*  $s_2 = t_1 = 0$ , the aggregate labour supply effect reads as  $(\eta_2 - \eta_1(q_2/q_1))l_2$ . This effect may have either sign. To assess the effect of a discrete reform, we must evaluate the expression in equation (19) in case  $s_2, t_1 > 0$ . This yields an effect that is smaller than  $(\eta_2 - \eta_1(q_2/q_1))l_2$ . This can be explained from two things. First,  $dt_1/ds_2$  is increasing in both  $s_2$  and  $t_1$ . Hence, if  $s_2, t_1 > 0$ , the weight of the tax rate increase and its corresponding negative effect upon labour supply is larger and the weight of the subsidy rate increase and its positive labour supply effect smaller than if  $s_2 = t_1 = 0$ . Second, if  $s_2, t_1 > 0$ , the tax rate increase implies a larger price change than if  $s_2 = t_1 = 0$  given that the initial price is proportional with  $1/(1-t_1)$ . Conversely, if  $s_2, t_1 > 0$ , the increase in the subsidy rate implies a smaller price change than if  $s_2 = t_1 = 0$  given that the initial price is proportional with  $1/(1+s_2)$ . For both reasons, young workers reduce their labour supply more than old workers increase theirs when the government increases the extent of front-loading of contributions. Still, no matter how large the discrete policy reform, the sign of the aggregate labour supply effect remains ambiguous as long as we do not restrict elasticities and productivity levels.

## 5. General taxation

In the previous section, the DB pension scheme was introduced in a first-best world: apart from DB pensions, no other distortions prevailed. In this section, we add a distortion to the

model. In particular, we assume that the government, apart from organizing a pension scheme, levies taxes in order to finance a certain amount of government consumption. Period-1 and period- 2 labour income serve as the tax base. To distinguish it from the implicit pension tax and subsidy, we will refer to this tax as a general tax. As we will see, adding this element that has a close counterpart in reality may turn the conclusions of the previous section on its head.

Let us use  $r$  to denote the rate of general taxation and  $R$  to denote the amount of public consumption. The expressions for the wage rate and leisure in period 1 and 2 are natural generalizations of the corresponding equations in the previous section:

$$w_1 = q_1(1 - r - t_1) \quad (20)$$

$$w_2 = q_2(1 - r + s_2) \quad (21)$$

$$v_1 = x_1(q_1(1 - r - t_1)) \quad (22)$$

$$v_2 = x_2(q_2(1 - r + s_2)) \quad (23)$$

The pension scheme budget constraint (equation (15)) applies as before. Total differentiation of this equation, using equations (20) to (23), gives us an expression for  $dt_1/ds_2$  which is a natural extension of the one in equation (16):

$$\frac{dt_1}{ds_2} = \frac{q_2 l_2 \left(1 + \frac{s_2}{(1-r+s_2)} \eta_2\right)}{q_1 l_1 \left(1 - \frac{t_1}{(1-r-t_1)} \eta_1\right)} \quad (24)$$

Next to this pension scheme budget constraint, we now have a government budget constraint, which relates the rate of general taxation to the financing requirement  $R$ :

$$R = q_1 r(1 - v_1) + q_2 r(1 - v_2) \quad (25)$$

Substituting equations (22) and (23) into equation (25) yields an equation that is a function of three policy instruments, namely  $t_1$ ,  $s_2$  and  $r$ :

$$R = q_1 r \left( 1 - x_1(q_1(1 - r - t_1)) \right) + q_2 r \left( 1 - x_2(q_2(1 - r + s_2)) \right) \quad (26)$$

Total differentiation of equation (26) gives an expression for  $dr/ds_2$  in terms of  $dt_1/ds_2$ . This can be further elaborated by substituting the expression for  $dt_1/ds_2$  that we derived in equation (24).

$$\begin{aligned} \frac{dr}{ds_2} &= \left\{ \frac{q_1 l_1 \left( \frac{r}{1-r-t_1} \right) \eta_1 \left( \frac{dt_1}{ds_2} \right) - q_2 l_2 \left( \frac{r}{1-r+s_2} \right) \eta_2}{q_1 l_1 \left( 1 - \left( \frac{r}{1-r-t_1} \right) \eta_1 \right) + q_2 l_2 \left( 1 - \left( \frac{r}{1-r+s_2} \right) \eta_2 \right)} \right\} \\ &= q_2 l_2 \left\{ \frac{\left( \frac{r}{1-r-t_1} \right) \eta_1 \left( \frac{1 + \frac{s_2}{(1-r+s_2)} \eta_2}{1 - \frac{t_1}{(1-r-t_1)} \eta_1} \right) - \left( \frac{r}{1-r+s_2} \right) \eta_2}{q_1 l_1 \left( 1 - \left( \frac{r}{1-r-t_1} \right) \eta_1 \right) + q_2 l_2 \left( 1 - \left( \frac{r}{1-r+s_2} \right) \eta_2 \right)} \right\} \end{aligned} \quad (27)$$

The denominator of both ratios at the RHS of equation (27) equals  $dR/dr$ , the tax revenue effect of a marginal change in the general tax rate. As before, we will assume it is positive (the economy is on the left side of the respective Laffer curve).

In order to explore the welfare effect of front-loading in the more general model, we need to derive anew an expression for  $dU/ds_2$ . This now reads as  $dU/ds_2 = \partial U/\partial s_2 + \partial U/\partial t_1(dt_1/ds_2) + \partial U/\partial r(dr/ds_2)$ . Upon using the information in equations (24) and (27), this can be elaborated as follows:

$$\begin{aligned} \frac{dU}{ds_2} &= q_2 l_2 - q_1 l_1 \frac{dt_1}{ds_2} - (q_1 l_1 + q_2 l_2) \frac{dr}{ds_2} \\ &= q_2 l_2 \left( 1 - \left( \frac{1 + \frac{s_2}{(1-r+s_2)} \eta_2}{1 - \frac{t_1}{(1-r-t_1)} \eta_1} \right) \right) \\ &\quad - (q_1 l_1 + q_2 l_2) q_2 l_2 \left\{ \frac{\left( \frac{r}{1-r-t_1} \right) \eta_1 \left( \frac{1 + \frac{s_2}{(1-r+s_2)} \eta_2}{1 - \frac{t_1}{(1-r-t_1)} \eta_1} \right) - \left( \frac{r}{1-r+s_2} \right) \eta_2}{q_1 l_1 \left( 1 - \left( \frac{r}{1-r-t_1} \right) \eta_1 \right) + q_2 l_2 \left( 1 - \left( \frac{r}{1-r+s_2} \right) \eta_2 \right)} \right\} \end{aligned} \quad (28)$$

Similarly, we derive the aggregate labour supply effect that applies in this section's model:

$$\begin{aligned}
& \frac{d(l_1+l_2)}{ds_2} \\
&= \left( \frac{\eta_2 l_2}{1-r+s_2} \right) - \left( \frac{\eta_1 l_1}{1-r-t_1} \right) \frac{dt_1}{ds_2} - \left( \frac{\eta_1 l_1}{1-r-t_1} + \frac{\eta_2 l_2}{1-r+s_2} \right) \frac{dr}{ds_2} \\
&= \left( \frac{\eta_2 l_2}{1-r+s_2} \right) - \left( \frac{\eta_1 l_1}{1-r-t_1} \right) \left( \frac{q_2 l_2}{q_1 l_1} \right) \left\{ \frac{1+\frac{s_2}{(1-r+s_2)}\eta_2}{1-\frac{t_1}{(1-r-t_1)}\eta_1} \right\} \quad (29) \\
&- \left( \frac{\eta_1 l_1}{1-r-t_1} + \frac{\eta_2 l_2}{1-r+s_2} \right) q_2 l_2 \left\{ \frac{\left( \frac{r}{1-r-t_1} \right) \eta_1 \left\{ \frac{1+\frac{s_2}{(1-r+s_2)}\eta_2}{1-\frac{t_1}{(1-r-t_1)}\eta_1} \right\} - \left( \frac{r}{1-r+s_2} \right) \eta_2}{q_1 l_1 \left( 1 - \left( \frac{r}{1-r-t_1} \right) \eta_1 \right) + q_2 l_2 \left( 1 - \left( \frac{r}{1-r+s_2} \right) \eta_2 \right)} \right\}
\end{aligned}$$

We now have all ingredients we need to explore the effects of frontloading in this more general model. In particular, we will focus upon two special cases. The first is where the two labour supply elasticities in the model are equal to each other. In the second, the period-2 labour supply elasticity is strictly larger than the period-1 labour supply elasticity. Let us start with the first case. We make two propositions.

*The case of equal labour supply elasticities*

**Proposition 3:**

If the government organizes pension contribution policies and general tax policies and if the period-1 and period-2 labour supply elasticities  $\eta_i$   $i = 1,2$  are equal to one another, the introduction of a marginal degree of front-loading in pension contributions exerts no effect upon welfare.

**Proof:**

Substitute  $\eta_1 = \eta_2 = \eta$  and  $s_2 = t_1 = 0$  into equation (28) and derive that  $dU/ds_2 = 0$ .

Somewhat paradoxically, the result of proposition 1 extends to this more general model, although now, different from the model to which proposition 1 applies, initial aggregate

labour supply is sub-optimally low due to general taxation. The effect of the reform upon aggregate labour supply is generally non-zero, like in the previous section:  $d(l_1 + l_2)/ds_2 = (-\eta_1(q_2/q_1) + \eta_2)l_2/(1 - r)$  (this can be derived by substituting  $\eta_1 = \eta_2 = \eta$  and  $s_2 = t_1 = 0$  into equation (29)). As in the previous section, the effect can have either sign. The only difference with the result in the previous section is that now the effect is a factor  $1/(1 - r)$  larger. The solution to the paradox can be found in the tax base effect. This tax base effects reads as :  $d(q_1l_1 + q_2l_2)/ds_2 = (-\eta_1q_2l_2 + \eta_2q_2l_2)/(1 - r)$ . With equal labour supply elasticities, this effect is zero. Hence, there cannot be any change in the rate of general taxation and the welfare effect is identical to that the previous section: zero.

**Proposition 4:**

If the government organizes pension contribution policies and general tax policies and if the period-1 and period-2 labour supply elasticities  $\eta_i$   $i = 1,2$  are equal to one another, the introduction of a discrete degree of front-loading in pension contributions is strictly welfare-decreasing.

**Proof:**

In order to validate proposition 4, we again decompose the introduction of front-loading into a series of many steps, in each of which  $s_2$  exhibits a very small increase. The welfare effect of the increase in  $s_2$  in the first step is zero (see proposition 3). To evaluate the welfare effect of the increase in  $s_2$  in the second step, write  $dU/ds_2$  in equation (28) in the following way:

$$\frac{dU}{ds_2} = q_2l_2 \left( 1 - \left( \frac{1 + \frac{s_2}{(1-r+s_2)}\eta_2}{1 - \frac{t_1}{(1-r-t_1)}\eta_1} \right) \right) - (q_1l_1 + q_2l_2) \frac{dr}{ds_2} \quad (30)$$

The RHS of equation (30) consists of two terms, both of which are zero in case  $\eta_1 = \eta_2 = \eta$  and  $s_2 = t_1 = 0$ . If we change this into  $\eta_1 = \eta_2 = \eta$  and  $s_2, t_1 > 0$ , the first term becomes negative. What happens to the second term? Recall the expression for  $dr/ds_2$  in equation (27). The numerator of the ratio on the RHS – zero in case  $\eta_1 = \eta_2 = \eta$  and  $s_2 = t_1 = 0$  – is positive in case  $\eta_1 = \eta_2 = \eta$  and  $s_2, t_1 > 0$ . Going back to equation (30), the second term on the RHS thus also turns negative. Taken together, it follows that  $dU/ds_2$  is negative if  $s_2, t_1 > 0$ . As in proposition 2, the result that the welfare effect of the increase in  $s_2$  in the second step is negative, applies to all following steps. Hence, we conclude that the introduction of a discrete degree of front-loading of pension contributions is welfare-reducing.

If we compare the results of a discrete policy reform for the more general model of this section with those for the more stylized model of the previous section, we note two differences. First, the reform reduces welfare more than before – even when the rate of

general taxation does not change. The reason is that the reform starts in an already distorted economy. The implicit tax and subsidy that are implied by the reform bite more because they come on top of a general tax on labour income. Second, the reform now reduces the tax base for the general tax – the implicit tax reduces tax revenues more than the implicit subsidy reduces revenues. The reason is the effect upon aggregate labour supply that becomes more negative the higher the degree of frontloading. The reduction of the tax base requires the government to raise the rate of general taxation in order to meet the financing requirement. This aggravates the initial distortion and implies a further welfare loss.

*The case of differential labour supply elasticities*

Let us now move on to the second case. In this case, the period-2 labour supply elasticity is larger than the period-1 labour supply elasticity, *i.e.*  $\eta_2 > \eta_1$ . Let us again focus first on the effects of a marginal policy reform. We can use equations (29), (27) and (28) to see what are the effects of this reform upon aggregate labour supply, the general tax rate and welfare. For the case of a marginal reform, these three equations reduce to the following,

$$\frac{dr}{ds_2} = - \left\{ \frac{q_2 l_2 \tilde{r}}{q_1 l_1 (1 - \tilde{r} \eta_1) + q_2 l_2 (1 - \tilde{r} \eta_2)} \right\} (\eta_2 - \eta_1) \quad (31)$$

$$\frac{d(l_1 + l_2)}{ds_2} = \frac{l_2}{1 - r} (-\eta_1 (q_2/q_1) + \eta_2) \quad (32)$$

$$\frac{dU}{ds_2} = \left\{ \frac{(q_1 l_1 + q_2 l_2) q_2 l_2 \tilde{r}}{q_1 l_1 (1 - \tilde{r} \eta_1) + q_2 l_2 (1 - \tilde{r} \eta_2)} \right\} (\eta_2 - \eta_1) \quad (33)$$

where  $\tilde{r}$  is a short-hand notation for  $r/(1 - r)$ . Equation (32) shows that the aggregate labour supply is proportional with  $(-\eta_1 (q_2/q_1) + \eta_2)$ . Hence, as before, the labour supply effect can have either sign, also when  $\eta_2 > \eta_1$ . Only in case of equal productivity levels, will the labour supply effect be strictly positive if the labour supply elasticity in period 2 exceeds that in period 1. As equation (31) shows, frontloading policies are proportional with  $\eta_2 - \eta_1$  and they unambiguously reduce the general tax rate when  $\eta_2 - \eta_1 > 0$ . What about welfare? Welfare is also proportional with the difference between the two elasticities and increases unambiguously. Although there are two factors that impinge upon welfare, only one is relevant in the case of a marginal reform. It is the expansion of the tax base, followed by a reduction of the general tax rate, that is fully responsible for the increase in welfare.

**Proposition 5:**

If the government organizes pension contribution policies and general tax policies and if the period-1 and period-2 labour supply elasticities  $\eta_i$   $i = 1,2$  obey the condition  $\eta_2 > \eta_1$ , the introduction of a marginal degree of front-loading in pension contributions is welfare-increasing.

The case of a discrete policy reform if  $\eta_2 > \eta_1$  is too complex to be solved analytically. Therefore, we will evaluate this more general case numerically in the next section.

## 6. Numerical analysis

To assess the more general case, we adopt a numerical version of the model developed above. In particular, we assume that preferences for leisure are of the isoelastic type:  $u_i(v_i) = -(1 - v_i)^{1+\gamma_i}/(1 + \gamma_i)$  for  $i = 1,2$ . This specification features a constant uncompensated labour supply elasticity,  $1/\gamma_i$ . From this we can derive the equations for leisure:  $v_i = 1 - w_i^{1/\gamma_i}$  for  $i = 1,2$ . The productivity levels  $q_i$  are specified as  $\bar{q}^{\gamma_i}$  for  $i = 1,2$ . This implies that the labour supply of young and old workers will be equally large in the case of zero taxation ( $t_1 = s_2 = r = 0$ ).

We solve the model in two steps. First, we solve the core of the model, consisting of the two equations for leisure, (22) and (23), the two price of leisure equations, (20) and (21), the pension scheme budget constraint, (15), and the government budget constraint, (26). This core model is a nonlinear system of six equations in six unknowns,  $v_1, v_2, w_1, w_2, t_1$  and  $r$ . Second, given the solution of these six variables, we use the household budget constraint, (8), and the utility function, (7), to derive the corresponding solutions for  $c_3$  and  $U$  respectively. All the solutions relate to two parameters, namely  $\gamma_i$   $i = 1,2$ , and four exogenous variables, namely  $s_2$  (the degree of frontloading),  $R$  (the government financing requirement) and the productivity levels of young and for old workers,  $q_i$   $i = 1,2$ .

We report on three groups of simulations. Each group in turn contains three simulations in which one parameter takes three different values. In the first group, the parameter *Ratio*, which is defined as the ratio between the uncompensated labour supply elasticities of old and young workers, takes values of 1.0, 2.5 and 5.0 respectively. In the second group, the parameter  $R$ , the financing requirement of the government, is varied. It takes value of 0, 0.05 and 0.10 respectively. Finally, the third group of simulations varies the value of  $\gamma_1$ , the reciprocal of the uncompensated labour supply elasticity. The values of  $\gamma_1$  are 1.0, 2.0 and 3.0 respectively; the implied values of the uncompensated labour supply elasticity of young workers are 1.0, 0.5 and 0.33 respectively.

We stress that all simulations are for illustrative purposes. Given the stylized nature of the model, it is not possible to choose parameter values such as to produce a realistic representation of reality. However, the simulations we report on are not arbitrarily chosen. They cover a range of cases with very different welfare effects. In addition, the values of the

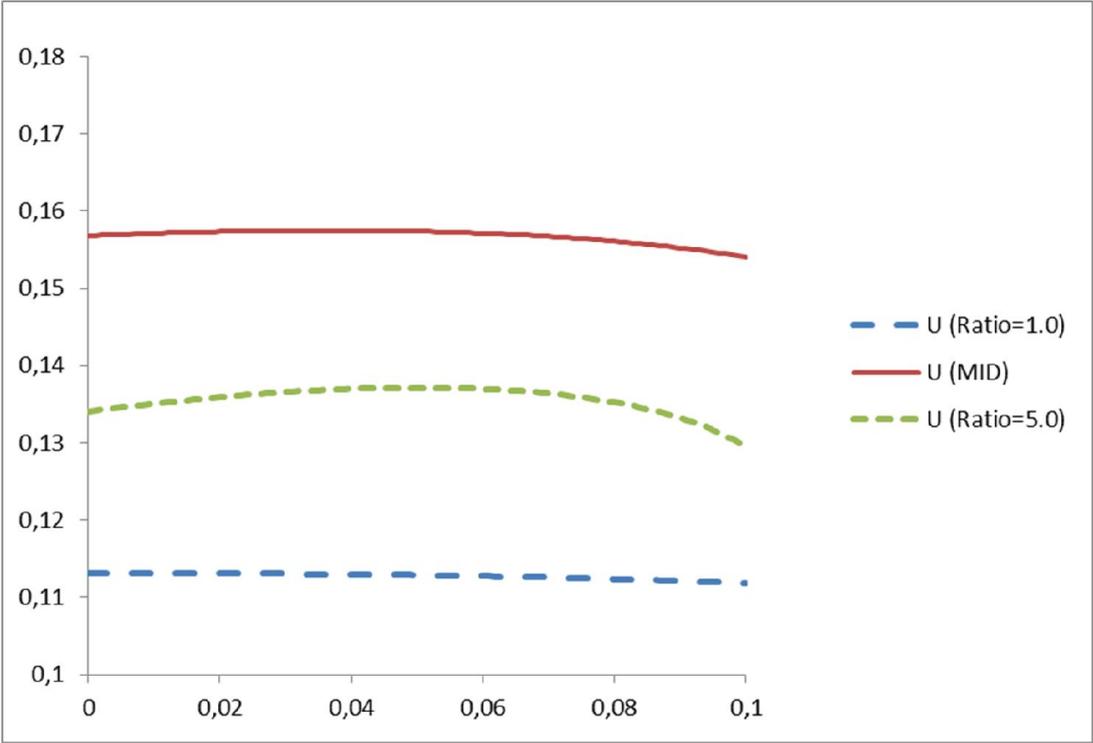
labour supply elasticities of the two groups of workers and the tax rates in the simulations are in the ranges found in the literature and in the data.

In all simulations, the parameter  $s_2$ , which measures the degree of frontloading, runs from zero (actuarially fair contributions) to 10 percent. The latter value should be high enough to cover real-world cases (the value for this variable that can be calculated using the data underlying Figure 2 is 6 percent). The range of values for  $s_2$  on which the simulations report, deserves also a comment. That is that a country does not choose a certain value for  $s_2$ . A country that chooses for the financing principle of frontloaded pension contributions chooses to have a non-zero value for  $s_2$ . How large  $s_2$  will be is implied by the rate of return on savings and the rate of productivity growth, both of which are variables that in general are difficult to monitor.

Table 1: Parameter configuration simulations Figure 3

Parameter	Input			Output	
	<i>Ratio</i>	<i>R</i>	$\gamma_1$	$\eta_1$	$\eta_2$
Value	1.0	0.05	2.0	0.50	0.50
	2.5				1.25
	5.0				2.50

Figure 3: Welfare effects of frontloading; the case of different values for *Ratio*

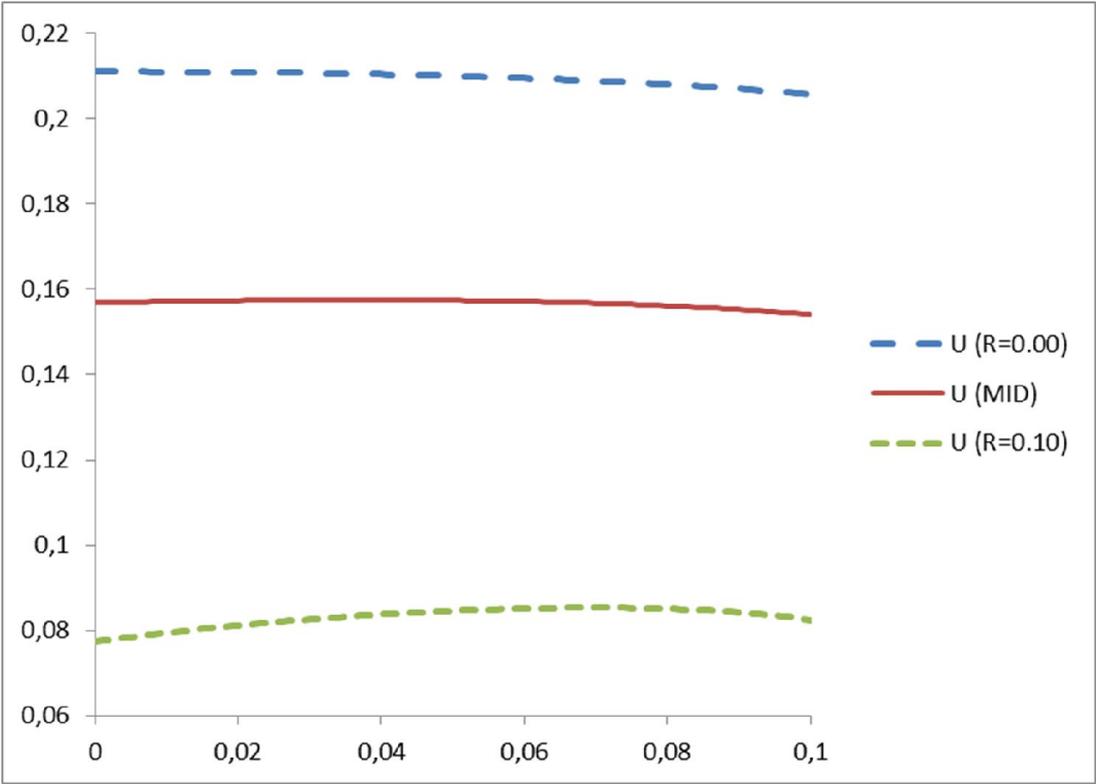


Let us start with Figure 3. It displays the welfare effects of different degrees of frontloading for three different values for *Ratio*. Table 1 shows the corresponding parameter configuration. The simulation in which *Ratio* equals one (represented by the blue, dashed curve) stands out. In this case, frontloading is negative for welfare, as we have derived in the previous sections; the curve is decreasing for all values of  $s_2$  between zero and 10 percent. The simulation in which *Ratio* equals 2.5 (represented by the red, solid curve) is denoted as MID. It contains middle values for all three variables; it is displayed in all three figures. Now, welfare is increasing for small values of  $s_2$ . For  $s_2$  equal to 3.6 percent, welfare is at the top. Beyond this point, welfare is a decreasing function of  $s_2$ . The simulation in which *Ratio* equals 5.0 (represented by the green, dotted curve) is the most dynamic one. The relationship between  $s_2$  and welfare is as in the case where *Ratio* equals 2.5, but now more pronounced. The turning point for  $s_2$  is the value of 5 percent. For lower values, the curve is increasing, for higher values decreasing.

Table 2: Parameter configuration simulations Figure 4

Parameter	Input			Output	
	<i>Ratio</i>	<i>R</i>	$\gamma_1$	$\eta_1$	$\eta_2$
Value	2.5	0.00 0.05 0.10	2.0	0.50	1.25

Figure 4: Welfare effects of frontloading; the case of different values for *R*

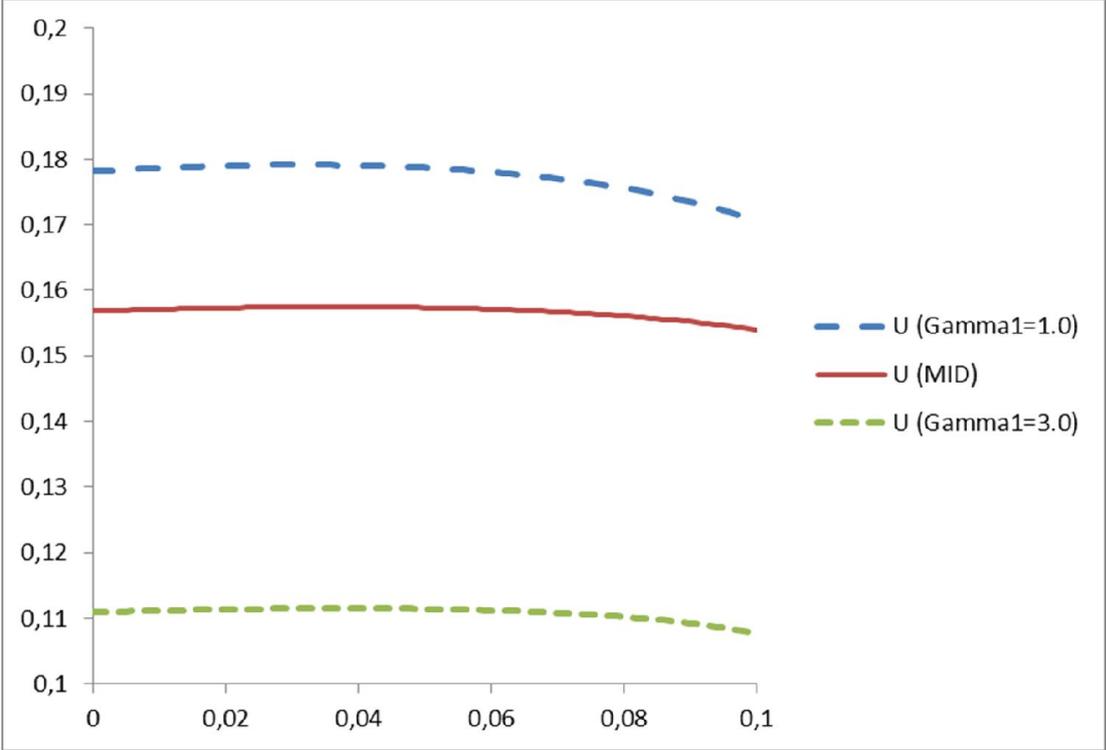


The second group of simulations is summarized in Table 2 and Figure 4. Here, it is the government financing requirement  $R$  that takes three different values: 0.00, 0.05 and 0.10. The blue, dashed curve represents the case with  $R$  equal to zero. This reflects the model studied in section 4. Welfare is decreasing for all values of  $s_2$  between zero and 10 percent. The red, solid curve reflects the MID case which we already have seen in our discussion of the first group of simulations. The green, dotted curve represents the case where  $R$  is 10 percent. This value is at the high end of the range of realistic values: it corresponds to a general tax rate that runs from 40.7 percent ( $s_2=0.00$ ) to 36.9 percent ( $s_2=0.10$ ). Of the three cases in this figure, it is the simulation with the largest initial distortion. This is reflected in the fact that welfare is increasing over a long range, *i.e.* until  $s_2$  assumes the value of 0.70. Only beyond  $s_2=0.70$ , welfare is decreasing. As a consequence, even at the upper end of the  $s_2$  range ( $s_2$  equals 0.10), is welfare higher than in the case without frontloading.

Table 3: Parameter configuration simulations Figure 5

Parameter	Input			Output	
	Ratio	$R$	$\gamma_1$	$\eta_1$	$\eta_2$
Value	2.5	0.05	1.0	1.00	2.50
			2.0	0.50	1.25
			3.0	0.33	0.83

Figure 5: Welfare effects of frontloading; the case of different values for  $\gamma_1$



The third group of simulations is summarized in Table 3 and Figure 5. Now, the parameter  $\gamma_1$ , which governs the labour supply elasticity of young workers and, given the value for *Ratio*, that for old workers as well, takes three different values: 1.0, 2.0 and 3.0. The blue, dashed curve represents the case with  $\gamma_1$  equal to 1.0. The corresponding labour supply elasticity for young workers is on the high side of estimates for this elasticity: 1.0. The elasticity for old workers is a factor 2.5 higher. The red, solid curve reflects the MID case which we already have encountered twice. The green, dotted curve represents the case where the uncompensated labour supply elasticity of young workers is 0.33 only and that of old workers is 0.83. The three simulations are relatively similar as regards the shape of the effect of  $s_2$  upon welfare. The points at which welfare becomes a decreasing function of  $s_2$  are 3.2 percent, 3.6 percent and 3.8 percent respectively. In all three cases is welfare at the upper end of the  $s_2$  range ( $s_2=0.10$ ) lower than at the lower end ( $s_2=0.00$ ).

What do we learn from these numerical simulations of the more general case? The simulations are based on a stylized model of labour supply behaviour. Therefore, I do not want to put too much emphasis on quantitative estimates of economic or welfare effects. This does not feel like a loss; the qualitative aspects are interesting enough. Indeed, the simulations show clearly that welfare may benefit from or drop down because of frontloading. In the polar cases with equal labour supply elasticities for young and old workers and with a zero government financing requirement, frontloading implies a lower level of welfare. On the other hand, if old worker labour supply is more price-elastic than young worker labour supply, the policy of frontloading may be welfare-increasing and the more so, the higher is difference in elasticities. As regards the government financing requirement, the same can be said. We want to stress again the nature of the policy problem: the government can choose to finance pension on the principle of actuarial fairness the ( $s_2=0$ ) of that of frontloading ( $s_2>0$ ). But the government cannot choose the value of  $s_2$ . This renders the welfare effect of frontloading uncertain; in some simulations, even the sign of the effect is unknown so long as the value of  $s_2$  is unclear. We therefore do not conclude that the scheme of frontloading is welfare-improving. Rather, we conclude that the scheme is not so bad for welfare as an analysis based on the labour market distortions of implicit pension taxes and subsidies alone would suggest.

## 7. Concluding remarks

Our result that frontloading may imply a lower welfare loss or even a welfare gain hinges upon two factors. One is the assumption that the labour market is distorted initially. The other is that old workers will react to the change in financial incentives to withdraw from the labour market.

As to the labour market, there is ample evidence that it is distorted. Unemployment is high in many countries, in absolute terms and from a historical perspective. Further, that taxes and social security contributions are high does not apply everywhere, but does seem to apply to many European countries. As to the price sensitivity of labour supply, there is a bit of uncertainty. On the one hand, there is plenty evidence that labour supply is price-sensitive, particularly at older ages. There is also substantial evidence that policy reforms that increase the degree of actuarial fairness of future pensions change worker's retirement behaviour. On the other hand, the idea that people discount the future effects of current decisions to a substantial degree has also firm ground (Frederick *et al.*, 2002). This is in line with survey

results that people's knowledge of their pensions is poor (Van Rooij *et al.*, 2012). One might wonder whether this depends on the frequency with which pension funds inform their members about the financial aspects of their pension. Particularly, an obvious idea is that frequent dissemination of information in understandable words could increase the impact of financial incentives. This idea is challenged by empirical estimates, however. Mastrobuoni (2011) fails to find a significant effect of additional information about pensions on the impact of retirement incentives upon workers' retirement behaviour.

One way to reconcile the conflicting types of evidence could be to assume that workers do take into account the future consequences of their decisions, but only after substantial discounting. The consequence of this line of thought is that young workers for whom pensions will occur only in the distant future would be little responsive to policies that affect pension rights, as future pensions are discounted for many years. However, old workers who are relatively close to retirement would account for the effects on their pensions as the discounting of future pensions has a smaller weight. In terms of our model, one could imagine that the labour supply of young workers features a zero elasticity and that of old workers a positive one. The results of our analysis would not change qualitatively, however. The implicit tax that is inherent with frontloading policies would vanish, whereas the implicit pension subsidy would remain. Hence, the scheme of frontloading contributions would continue to increase the labour supply of older workers. Again, this would be welfare-reducing in case the labour market would not be subject to other types of distortions; it could be welfare-enhancing if other distortions implied that aggregate labour supply was sub-optimally low initially.

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